Optimization System for Train Set Planning of Inter-City Rapid Trains with the Application of the Flexible Seat Class Assignment Approach

Munenori SHIBATA*, Shintaro TERABE**, Hisao UCHIYAMA**
* Railway Technical Research Institute, Tokyo, JAPAN
** Tokyo University of Science, Chiba, JAPAN

Abstract

Most of intercity express trains in Japan supply different seat classes (types) in commonly used ordinary (coach class) cars, (1) reserved seats, where passengers are completely guaranteed to have their seating by paying additional fare for seat reservation in each boarding, (2) non-reserved seats, where passengers can sit anywhere if seats are available. The number of cars in one train set, the split of respective seat types and additional fare for seat reservation are usually fixed year-round based on the empirical knowledge. Such a current seat type assignment approach actually causes inefficient utilization of train seats: for example, non-reserved seats are highly congested while there are still vacant seats in reserved seats. Such a situation also decreases profitability of privatized railway companies. The study tries to develop an optimization system for train set planning with the application of the flexible seat type assignment approach proposed by the authors through some case studies. The study finds that the developed optimization system is effective in: (1) improving the efficiency of the facility utilization, (2) increasing the utility of passengers by decreasing the number of reservation request rejection and decreasing load factor of non-reserved seats, and (3) increasing revenue of railway companies.

Keyword
Inter-city Rapid Train, Revenue Management, Flexible Seat Class (Type) Assignment, Genetic Algorithm

1. Introduction

In many countries, inter-city rapid railway network is highly expected to play an important role in developing low carbon society, through the potential of its high efficiency of energy consumption per passenger kilometer. Additionally, because many train operating organizations are privatized, such private companies have responsibility to supply transportation services sustainably to travelers based on the adequate revenue. Therefore, increasing the utilization and profitability of inter-city rapid trains is one of the important issues.

Revenue management is one of the major approaches for solving these issues. Some studies to apply revenue management to inter-city rapid trains have been conducted and demonstrated some effectiveness (e.g. Ciancimino et al. (1999), Suh et al. (2001), Terabe et al. (2006), Bharill et al. (2008)). In aforementioned studies, unfortunately, taking consideration of the individual passenger behaviors such as mode choice, fare type choice, seat type choice, etc. has not been tried. On the other hand, Ongprasert, S. (2006), Terabe et al. (2007) have revealed that the demand estimation based on passenger behaviors can improve the capability of the seat inventory control,
which is a major approach of revenue management. Therefore, the flexible seat type assignment is also expected to be more efficient by applying individual behavior models.

Most of inter-city rapid trains have several kinds of seat classes. For example, most Japanese inter-city rapid trains have two seat types in commonly used ordinary (coach class) cars: (1) reserved seats, where to seat is fixed and passengers are completely guaranteed to have their seating during the trip. Passengers must pay around 500 yen (equivalent to about US$5, 5 euros) in each boarding as an additional fare for a seat reservation. (2) non-reserved seats, where to seat is not fixed, passengers can sit anywhere if seats are available. It means passengers take a risk of standing on trains if non-reserved seats are fully occupied. In most cases, the number of cars in one train set, the split of respective seat types and additional fare for seat reservation are usually fixed year-round based on the empirical knowledge. Such a current seat type assignment causes inefficient utilization: for example, non-reserved seats are highly congested while there are still vacant seats in the reserved seats. In order to improve the utilization of facilities, the split of seat classes (types) and the additional fare settings for a seat reservation should be arranged flexibly with responding to the passengers’ demand.

Thus, a seat type choice behavior model is calibrated by the authors based on the individual trip survey (Shibata et al. (2010)). Subsequently, the “flexible seat type assignment system” is developed and some effectiveness for improving profitability and passenger utility are illustrated quantitatively by the authors, where the seat type choice behavior model is applied to demand estimation of each seat type (Shibata et al. (2010), Shibata (2010), Shibata (2011)). In the abovementioned system, the objective function is the maximization of load factor of reserved seats, control variables are additional fare for a seat reservation, the number of supply of each seat type and selling ratio of reserved seats to the estimated reservation requests for reserved seats. On the other hand, the number of cars in one train set is assumed to be fixed. Therefore, it is impossible to apply the flexible seat type assignment system to train set planning. The study tries to improve the “flexible seat type assignment system” in order to optimize the train set planning with the application of effective flexible seat type assignment approach.

2. Demand Prediction Model

As mentioned, a seat type choice behavior model is calibrated by the authors based on the individual trip survey (Shibata et al. (2010)). This calibrated model is installed to the train set optimization system developed in the study in order to predict demand of each seat type when the additional fare for seat reservation is changed from the actual price. The model is based on the disaggregated choice behavior model, usually applied to mode choice behavior, route choice behavior, etc. in the fields of transportation demand analyses and demand predictions.

Initial findings of the individual trip survey suggest that it is difficult to describe such a choice behavior not only by travel time on board, some trip attributes, such as the number of accompanies and its category. On the contrary, magnitude of importance on some preference factors of each seat type, such as “complete seating during the trip”, have high influence on such a choice behavior. Hence, the disaggregate choice behavior model is calibrated with the consideration of the heterogeneity of passengers’ preference, explained by the magnitude of importance on some characteristics of each seat type.

Therefore, the seat type choice behavior model is composed of Multiple Indicator Multiple Cause (MIMIC) model (Figure 1), a kind of Structural Equation Modeling (SEM), and disaggregate logit model. The former model extracts some intangible latent factors of each passenger, which explain the heterogeneity of passengers’ preference for seat type choice.
The MIMIC model is one of the popular techniques to identify the relationship between observed importance on preference factors and some trip and socio-economic attributes with the intermediary of latent variables. The model is composed of a structural equation (1) and a measurement equation (2) formulated as follow:

\[ \mathbf{w} = \mathbf{Bs} + \zeta \]  
(1)

\[ \mathbf{y} = \mathbf{\Lambda w} + \mathbf{\epsilon} \]  
(2)

where
- \( \mathbf{w} \) = vector of latent variables
- \( \mathbf{s} \) = vector of attributes identifying \( \mathbf{w} \) in the structural equation
- \( \mathbf{y} \) = vector of magnitude of importance on preference factors
- \( \mathbf{B}, \mathbf{\Lambda} \) = vector of unknown parameters
- \( \zeta \) = random component of structural equation \( A/W \text{MVN}(0, \Psi) \)
- \( \mathbf{\epsilon} \) = random component of measurement equation \( A/W \text{MVN}(0, \Theta) \)

After estimating all unknown parameters in the MIMIC model and variance-covariance matrix, expected values of latent variables are calculated using equation (3) for each sample. Additionally, expected values of these latent variables are calculated by using only the structural equations in some virtual cases (Morikawa et al. (1993)).

\[ \hat{\mathbf{w}} = \hat{\mathbf{Bs}} + \hat{\mathbf{\Psi}} \hat{\mathbf{\Lambda}}^{-1} \left( \hat{\mathbf{\Lambda}} \hat{\mathbf{\Psi}} \hat{\mathbf{\Lambda}}' + \hat{\Theta} \right)^{-1} \left( \mathbf{y} - \hat{\mathbf{\Lambda}} \hat{\mathbf{Bs}} \right) \]  
(3)

Abovementioned expected latent values are installed to the utility function of the disaggregate logit model. Estimated parameters of the MIMIC model are shown in the Figure 2, and estimated parameters of the disaggregate seat type choice behavior models are shown in Table 1.
### Table 1: Estimated Parameters for Seat Type Choice Models

<table>
<thead>
<tr>
<th>Variables</th>
<th>Without latent variables</th>
<th>With latent variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel cost (10,000 yen)</td>
<td>$\alpha_1$ -2.891 (-1.46)</td>
<td>-10.85 (-4.23)</td>
</tr>
<tr>
<td>Travel time on board (hour)</td>
<td>$\alpha_2$ 0.727 (7.98)</td>
<td>0.438 (4.22)</td>
</tr>
<tr>
<td>Guarantee of seating (latent)</td>
<td>$\alpha_3$ ***</td>
<td>1.037 (11.9)</td>
</tr>
<tr>
<td>Flexibility of seating (latent)</td>
<td>$\alpha_4$ ***</td>
<td>0.894 (10.3)</td>
</tr>
</tbody>
</table>

| Row-bar (goodness of fit index) | 0.220 | 0.342 |
| AIC (Akaike’s Information Criteria) | 1275.09 | 1078.40 |
| Hit ratio (%) | 75.1 | 79.1 |
| Number of samples | 1176 | 1176 |

( ) : t-value

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**Figure 2 Estimated Parameters for the MIMIC Model**

GFI : 0.99  AGFI : 0.96  RMSEA : 0.040
N=1176

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**Table 2** Estimated Parameters for the MIMIC Model

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N=1176
Even the model without latent variables has the significant goodness-of-fit indicated by row-bar. On the other hand, the model with latent variables has more significant goodness-of-fit in comparison with the former. Moreover, the parameter of “travel cost” in the model with latent variables has become statistically significant. Thus, the introduction of heterogeneity of passengers’ preference makes the model stable and valid for describing seat type choice behavior. In the following optimization system, the model with latent variables is applied to estimate the choice probability of each seat type when the additional fare for seat reservation is changed from the actual price.

3. Development of the Train Set Planning Optimization System

3.1 Formulation

An optimization system for train set planning with the application of flexible seat type assignment approach is formulated as one of the mathematical optimization problems described below.

Firstly, the latent demand of OD (Origin-Destination) \( i \) on seat type \( j \) on train \( k \) is estimated. Equations (4) and (5) are utility functions of seat type choice behavior model and the choice probability of OD \( i \) on seat type \( j \) on train \( k \) is calculated by equations (6) and (7). It is assumed that the overall demand of OD \( i \) on train \( k \) is observed or estimated by observed demand data on every train accumulated everyday by railway companies’ seat reservation system, automatic ticket gates or estimation report of riding passengers by train conductors. Given \( OD_{ik} \) multiplied by \( P_{ijk} \) equals the latent demand of OD \( i \) on seat type \( j \) in train \( k \) (\( OD_{ijk} \)) shown in equation (8).

\[
U_{ik} = \alpha_1 \times fare_{ik} + \alpha_2 \times time_{ik} + \alpha_3 \times lat1_{ik} \\
U_{jk} = \alpha_1 \times fare_{jk} + \alpha_4 \times lat2_{ik} \\
\exp(U_{ijk}) = \frac{1}{1 + \exp(-\alpha_1 \times \Delta fare_{ik} + etc_{ik})} \\
P_{ijk} = 1 - P_{ijk} \\
OD_{ijk} = P_{ijk} \times OD_{ik}
\]

where

- \( fare_{ik} \) : fare of OD \( i \) when choosing seat type \( j \) on train \( k \) (same as travel cost)
- \( time_{ik} \) : travel time of OD \( i \) on train \( k \)
- \( lat1_{ik} \) : expected value of “guarantee of seating” of OD \( i \) on train \( k \)
- \( lat2_{ik} \) : expected value of “flexibility of seating” of OD \( i \) on train \( k \)
- \( \Delta fare_{ik} = fare_{ik} - fare_{jk} \) : additional fare for seat reservation of OD \( i \) on train \( k \)
- \( etc_{ik} = -\alpha_2 \times time_{ik} - \alpha_3 \times lat1_{ik} + \alpha_4 \times lat2_{ik} \)

As for reserved seats (seat type \( j=1 \)), \( SOD_{ikh} \) is the volume of reserved seat sales of train \( k \) on OD \( i \). Latent demand (in other words, the number of requests for seat reservation) \( OD_{ikh} \) times sales ratio for reservation requests \( \beta_{ik} \) equals to the volume of sales \( SOD_{ikh} \) as shown in equation (9) (Ongprasert, S. (2006)).

On the other hand, \( SOD_{ik} \) is the volume of non-reserved seat sales of train \( k \) on OD \( i \). \( SOD_{ik} \) is calculated by subtracting \( SOD_{ikh} \) from \( OD_{ik} \) as shown in equation (10), with assumption that if a reservation request for reserved seat on train \( k \) is rejected, he/she buys a non-reserved seat ticket on
the same train $k$. In other words, the formulation does not consider train choice behavior or trip cancellation behavior caused by rejection of seat reservation requests or high congestion in non-reserved cars. $SOD_{ik}$ is transformed to the volume of seat type $j$ of train $k$ on the station-to-station section $l$, shown as $SDSEC_{ik}$ (equation (11)).

\[
SOD_{i1k} = \beta_{ik} \times OD_{i1k} \\
SOD_{i2k} = OD_{ik} - SOD_{i1k} \\
SDSEC_{ijk} = \sum_l \delta_{il} \times SOD_{ijk}
\]

where

\[
\delta_{il} \text{ : when } SOD_{ik} \text{ passes section } l = 1, \text{ otherwise } = 0
\]

As for the volume of seat, $SUP_{ik}$ is defined as the number of type $j$ seats of train $k$ on section $l$. $CAR$ is the number of cars in one train set and $NUM$ is the number of seat in one train car. Therefore, $CAR$ multiplied by $NUM$ minus $SUP_{rik}$ is the number of non-reserved seats of train $k$ on section $l$. The objective function “ess” is defined as the sum of squares of the difference between $SUP_{ik}$ and $SDSEC_{ik}$. The difference between the volume of sales on each seat type and the number of seats of each type should be decreased in order to increase passenger load factor of each seat type and decreased the number of revealed seat reservation request rejections. Therefore, the objective function is to be minimized in this optimization problem. Finally, the mathematical optimization problem is formulated as shown in equation (12). $\beta_{ik}, \Delta fare_{ik}, SUP_{ik}$ and $CAR$ are controlled variables in this problem.

\[
\min_{\beta_{ik}, \Delta fare_{ik}, SUP_{ik}, CAR} \quad ess = \sum_k \sum_l \sum_j \left( SUP_{ijk} - SDSEC_{ijk} \right)^2 \\
= \sum_k \sum_l \left( \left( SUP_{i1k} - \sum_j \left( \frac{\delta_{jl} \times \beta_{ik} \times OD_{ik}}{1 + \exp(-\alpha_j \times OD_{ik} + etc_{ik})} \right) \right)^2 + \left( CAR \times NUM - SUP_{i1k} \right) - \sum_l \left( \delta_{il} \times \left( OD_{ik} - \frac{\beta_{ik} \times OD_{ik}}{1 + \exp(-\alpha_i \times \Delta fare_{ik} + etc_{ik})} \right) \right)^2 \right)
\]

subject to $SDSEC_{i1k} \leq SUP_{i1k}$ $0 \leq \beta_{ik} \leq 1$ $\Delta fare_{ik} \geq 0$

3.2 Development of the Train Set Planning Optimization System

As mentioned, this problem is assumed as a multimodal non-linear optimization problem, because the latent demand $OD_{ik}$ is changing non-linearly according to controlling $\Delta fare_{ik}$. Therefore, Genetic Algorithm (GA), one of the solving techniques derived from meta-heuristics optimization approach, is applied to develop the train set planning optimization system with the consideration of application of flexible seat type assignment approach.

The flow diagram of developed optimization system is shown in Figure 3. Firstly, some individuals including all control variables are generated by random numbers. Secondly, these individuals are evaluated by calculating objective function of each individual on the basis of demand estimation. In order to optimal-solution-search efficiently, the elitist strategy is applied. Genetic operations (Cross over and Mutation) are applied to non-elitist individuals to generate new individuals. New individuals are also evaluated by calculating objective function on the basis of demand estimation. Afterwards, new individuals and non-elitist individuals are selected stochastically according to evaluation rankings. Finally, the best individual among elite and selected individuals is recorded. When
the best individual does not evolve after 3,000 iterations, the system terminates the optimization calculation and outputs the best result.

Figure 3 Flow Diagram of Developed Optimization System

4. Case Study

This chapter illustrates the result of a case study of train set planning optimization with the application of flexible seat type assignment approach. Four inter-city rapid trains (Trains No.1~No.4) in a railway line are subjects of this case study, where overall OD data (OD$_{ik}$) are estimated by dividing actual observed reserved seat OD data (Ongprasert, S. (2006)) by estimated choice probability of reserved seats. Some assumptions are applied to this case study described below.
Assumptions

- Number of station is 4 (Stations A to D) and all objective trains are operated in down line.
- There are only commonly used ordinary (coach class) class cars. There are no green cars (first class cars).
- The number of seats is 56 in each train car.
- In almost rapid trains in Japan, additional fares for seat reservation are 310 yen (equivalent to US$3, 3 euros) in off season, 510 yen (equivalent to US$5, 5 euros) in regular season, 710 yen (equivalent to US$7, 7 euros) in peak season. Therefore, the additional fare for seat reservation is controlled in every train and every OD pair between 310 yen and 710 yen by every 10 yen (equivalent to 10 cents).
- The number of cars in one train set is consistent in each objective train.
- The number of cars in one train set is controlled between 5 and 7 cars.
- If a reservation requests for reserved seat on train \( k \) is rejected, he/she buys non-reserved seat ticket on the same train \( k \) (without the consideration of train choice behavior or trip cancellation behavior).
- There are no group passengers.

In order to reveal the effectiveness of flexible seat assignment approach (hereafter “flexible case”), a comparative case should be calculated simultaneously, where every train has 5 ordinary class cars (280 seats) and seats are evenly split to reserved and non-reserved seats (140 seats) which is the usual assignment in most rapid trains in Japan. The additional fare for seat reservation is 510 yen (equivalent to US$5, 5 euros) in every train in every OD. Actual ticket sales rule known as first-come-first-served is applied in this simulation case. This comparative case is named as “fix case” in the study (Figure 4).

![Fix case](image)

Table 2

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>**</td>
<td>510</td>
<td>510</td>
<td>510</td>
</tr>
<tr>
<td>A</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>510</td>
</tr>
<tr>
<td>B</td>
<td>**</td>
<td>**</td>
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<td>510</td>
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<td>C</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>510</td>
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<tr>
<td>D</td>
<td>**</td>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
</tbody>
</table>

*Seat map is from JR timetable

The simulation results in both flexible and fix cases are shown in Table 2 and the results of flexible seat type assignment in each train are illustrated in Figures 5-8. Average passenger load factor on reserved seats is improved. The flexible assignment approach increases revenue because of the abovementioned improved load factor and declined number of rejection for seat reservation requests, which means improved profitability of privatized railway companies. From the view points of passengers, declined number of rejection for seat reservation requests contributes to improving passengers’ comfort while they book. Moreover, improved congestion rate of non-reserved seats shown by load factor also leads to passengers’ comfort while travelling on board, because it is almost possible for passengers to get their seats in non-reserved cars and high congestions on non-reserved cars are resolved. Additionally, increased seat utilization shown by the load factors also means the improvements in the efficiency of energy consumption per passenger kilometer of railways.
Table 2 Simulation Results

<table>
<thead>
<tr>
<th>Index</th>
<th>Flexible case (A)</th>
<th>Fix case (B)</th>
<th>(A)-(B)</th>
<th>Determination</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;ess&quot; (objective function)</td>
<td>22,894</td>
<td>46,742</td>
<td>-23,848</td>
<td>Improved</td>
</tr>
<tr>
<td>Average passenger load factor in reserved seats</td>
<td>92.88%</td>
<td>92.56%</td>
<td>0.32%</td>
<td>Slightly improved</td>
</tr>
<tr>
<td>Average passenger load factor in non-reserved seats</td>
<td>100.59%</td>
<td>130.77%</td>
<td>-30.19%</td>
<td>Improved</td>
</tr>
<tr>
<td>Revenue (10,000 yen)</td>
<td>2,011,679</td>
<td>2,000,1470</td>
<td>11,5330</td>
<td>Improved</td>
</tr>
<tr>
<td>The number of rejection for seat reservation requests</td>
<td>79</td>
<td>444</td>
<td>-365</td>
<td>Improved</td>
</tr>
<tr>
<td>The number of cars in one train set</td>
<td>6</td>
<td>5</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5 the Results of Flexible Seat Type Assignment (Train No.1)

Figure 6 the Results of Flexible Seat Type Assignment (Train No.2)
5. Conclusion Remarks

The study tries to develop the optimization system of train set planning with the application of the flexible seat class assignment approach proposed by the authors through some case studies. The simulation finds that developed optimization system is effective in the following aspects simultaneously: (1) improving the efficiency of the facility utilization, (2) increasing the utility of passengers by decreasing the number of reservation request rejections and improving congestion rate of non-reserved seats, and the (3) increasing revenue for railway companies. Introduction of the developed system derived from revenue management research field to railway companies is expected to improve both the rolling stock investment planning and appropriate seat type assignment planning in everyday operation. Accordingly, it must contribute to improving sustainability of inter-city railway
operation by increasing profitability and passengers’ utility.

The study concludes that introducing revenue management consciousness into railway operation and upgrading the technique of service supply planning in order to adjust to characteristics of each country’s railway operation are important.

There are some further studies for improving the optimization system. For example, seat class (type) choice behavior model should include upper class seats (e.g. first class seats) as a choice alternative when there are some upper class seats. Modeling the train choice behavior including trip cancellation is also an important research issue for occasions that some reservation request rejections and high congestion in non-reserved cars are happened.

References