Challenge G: An even more competitive and cost efficient railway

Effect of parameter uncertainty on the numerical estimate of a railway vehicle critical speed

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Abstract

The paper describes a joint study carried out by SNCF and Dipartimento di Meccanica Politecnico di Milano, aimed at investigating how parametric uncertainty can be treated in the framework of virtual homologation of railway vehicles in respect to vehicle dynamics. In railway vehicle sources of parameter uncertainty may arise from inaccuracy in the modelling of a vehicle component or from a scatter in the behaviour of nominally identical components, on account of the variability implied by the component manufacturing process.

The approach proposed in this paper, completely new in the railway field, is to use statistical methods having different complexity (and entailing a proportional computational effort), to analyse the propagation of uncertainty from the parameters input in the vehicle mathematical model to the results of running dynamics, in terms of the assessment quantities used for vehicle homologation. The problem is treated by numerical means, being the dependency of simulation outputs from the input parameters typically non-linear, and not defined in an analytical form.

Keywords: railway virtual homologation, uncertainty in railway vehicle, Montecarlo simulation

1. Introduction

The process of vehicle homologation represents a heavy burden for rolling stock manufacturers as well as for operators, and significantly affects the cost and time to market the railway rolling stock. In particular, railway vehicle homologation in respect to running dynamics, almost completely relies at present on the use of physical tests, which not only are expensive and require a huge investment of time and effort, but also are affected by a number of uncontrolled variables, including the uncertainty of vehicle parameters. Therefore, railway operators and system integrators are considering the possibility to partially replace physical testing by virtual testing based on validated mathematical models. This new approach offers a possibility to reduce the amount of line testing and the associated cost and effort, and at the same time underpins a more objective homologation process, as it allows the investigation of issues not addressed by the physical homologation process, such as the impact of uncertainties in components behaviour.

Aim of this study, jointly carried out by SNCF and Politecnico di Milano, is to investigate how parametric uncertainty can be treated in the framework of virtual homologation of railway vehicles in respect to vehicle dynamics. Two sources of parameter uncertainty can be dealt with during virtual homologation: on one hand, parameter uncertainty may arise from inaccuracy in the modelling of a vehicle component which, at least to some extent, can be represented in the mathematical model of the vehicle as a deviation of model parameters from their ‘true’ value. On the other hand, a scatter in the behaviour of nominally identical components can be expected, on account of the variability implied by the component manufacturing process. This second effect cannot be quantified by means of line testing on one simple train set, but can be handled by the integration of computer aided homologation and appropriate statistical tools [1,2,3].

The approach proposed in this paper, completely new in the railway field, is to use statistical methods having different complexity [4] (and entailing a proportional computational effort), to analyse the propagation of uncertainty from the parameters input in the vehicle mathematical model to the results of running dynamics, in terms of the assessment quantities used for vehicle homologation. The problem is treated by numerical means, being the dependency of simulation outputs from the input parameters typically non-linear, and not defined in an analytical form.

The proposed paper presents the numerical methodology and its application to the numerical estimation of the critical speed for a railway vehicle, demonstrating the possibility to include parameter uncertainty effects in the virtual homologation of a railway vehicle in a simple and cost-effective way, taking advantage from the use of appropriate numerical tools.
The paper is organised as follows: Section 2 introduces the method used to estimate the vehicle critical speed by means of numerical simulation and describes how parametric uncertainty can be introduced in that calculation. Section 3 describes a simplified linearised approach, whereas in Section 4 more complex and complete methods are introduced to analyse the propagation of uncertainty. Sections 5 deals with the description of the results obtained when the methods introduced previously are applied to the calculation of the critical speed of a railway vehicle, considering the case of a locomotive car from a concentrated power EMU train. Finally, conclusions are drawn in Section 6.

2. Numerical estimate of the critical speed and the effect of parameter uncertainty

In order to assess the effect of parameter uncertainty, the rail vehicle is considered as a mixed probabilistic-deterministic process, and the vehicle critical speed $V$ is defined as a function of deterministic and random variables:

$$ V = f(u, p) $$

(1)

where $u$ is the vector of input variables affected by uncertainty, and $p$ is the vector of deterministic variables in the problem, whose values are assumed to be known without uncertainty. Since no analytical expression is available for the dependency of the critical speed on the uncertain and deterministic parameters expressed by equation (1), the critical speed is computed using numerical simulations performed on a multi-body vehicle model [5]. The vehicle considered is a concentrated power locomotive for high-speed train with maximum service speed 240 km/h, whose data do not correspond to an existing vehicle, but were properly chosen to define a realistic case study.

2.1 Calculation of the critical speed

To obtain an accurate numerical estimate of the critical speed for a railway vehicle, non-linear effects inherently associated with wheel/rail contact and the actual geometry of wheel and rail profiles have to be properly accounted for. The method used in this paper is based on the simulation of vehicle non-linear running behaviour in tangent track, subject to the random excitation produced by track irregularity [5,6], and is inspired by the prescriptions of the European standard EN14363 regarding the experimental assessment of the critical speed for a rail vehicle.

Numerical simulations are performed considering the straight track running of the vehicle in presence of random track irregularity, with stepwise increasing speed. For each vehicle speed considered, the r.m.s. of both track shift forces and lateral bogie acceleration (pass-band filtered as explained above) are extracted, and the process is iterated until one of the r.m.s. values exceeds the corresponding limit as stated by the standard. The critical speed value is then obtained by measured linear interpolation between the two speed values falling across the limit condition.

2.2 Vehicle parameters affected by uncertainty

In the case study presented in this paper, parametric uncertainty is considered to occur only on the secondary yaw damper parameters, which are modelled using a linear viscous dashpot with serial stiffness, and parameter uncertainty is assumed on the stiffness parameter $k_d$ and on the damping parameter $c_d$.

With respect to all parameters listed above, two cases are addressed in the paper: on one hand, parameter uncertainty may arise from inaccuracy in the modelling of a vehicle component, which means the ‘true’ value of the parameter is different from the one used in the simulation. In this case, the same uncertain value of the parameter is considered for all components of the same type in the bogie, i.e. for the left and right yaw damper, and the set of parameters affected by uncertainty is defined as follows:

$$ u = \{ k_d, c_d \} $$

(2)

The assumptions on the probabilistic distribution of parameters affected by uncertainty depend on the information available on the parameters themselves, such as statistical datasets and expert judgement. In this work, all statistical variables in vector $u$ are supposed to be represented by statistically independent Gaussian variables, with the mean value corresponding to the nominal value of the parameter and standard deviation defined so that the 0.15-99.85 percentiles correspond, in the analysed case, to a ±15% variation of the yaw damper characteristic with respect to the nominal value, which is in line with the tolerances normally set on the supply of this railway component.
3. Propagation of uncertainty: the ‘one at time’ variation method

The simplified method is based on the relationship between critical speed and input variability (eq. 4) obtained as the first order Taylor’s expansion of Equation (1) around the vehicle nominal condition (i.e nominal value \( u_0 \) of the uncertain parameters)

\[
V = V_0(u_0, p) + \sum \alpha_i \cdot \Delta u_i
\]  

(3)

\( V_0 \) is the vehicle critical speed in nominal configuration and \( \alpha_i \) are sensitivity coefficients expressing the linearised effect of parameter variations \( \Delta u_i \) on the critical speed. The method used to compute the sensitivity coefficients is based on the OAT (one at time) screening plan approach in which the impact of changing each parameter at time is evaluated [7]. The nominal values and two extreme values of each uncertain parameter are selected to define the effect of single parameters variation, and the sensitivity coefficients \( \alpha_i \) are defined using the least square method.

Assuming a Gaussian probability density distribution for the uncertain parameters, and using the linearised input-output relationship (4), the probability density distribution obtained for the vehicle critical speed is also Gaussian, with mean \( \mu_V \) and standard deviation \( \sigma_V \) defined according to fundamental statistics [2]:

Using the above results, the vehicle critical speed can be defined in probabilistic terms, e.g. as the 0.15 percentile of the probability density distribution defined by \( \mu_V \) and \( \sigma_V \), which would mean a 99.85% probability of meeting the homologation requirement on the critical speed.

4. Propagation of uncertainty: the “multiple tendency” method

A more comprehensive but also more complex approach to define the propagation of uncertainty from inputs to outputs, entails the simultaneous variation of all inputs, accounting as well for possible interactions between them. Consequently a non-linear dependence of the outputs from the uncertain input values [7] can be envisaged which is completely neglected if the OAT process is implemented. The process describes the input-output relationship through a calculation of output values corresponding to several input combinations defined according to statistical rules.

In its simplest form this process foresees that each input parameter is sampled several times to represent its real distribution according to its probabilistic characteristic defined as described in section 2. Then combining all the input parameters, several sets of input values are obtained, each one representing one “realisation” of the input-output problem. Solving the input-output problem for each realization (in this case by means of numerical simulation), the overall probabilistic characteristics of the output is identified.

Although its application is relatively simple, the process may become severely time consuming if a large amount of input parameters are considered.

In this work two different approaches have been analysed and compared in terms of efficiency and reliability: the first one is based entirely on Montecarlo simulation techniques (MCS in the following) [8], while the second one is built on a combination of Design of experiment theory [9] and the above Montecarlo simulation techniques (DOE& MCS in the following).

As far as the first method is analysed the following steps need to be fulfilled:

i. Random parameter values are generated for inputs (i.e stiffness and damping characterising the bogie yaw damper) according to their assumed probabilistic distribution;

ii. For each realisation of the random variables set defined at point i) the corresponding output (i.e. vehicle critical speed) is computed performing non linear simulation of the vehicle running behaviour.

iii. For the output sample (i.e critical speed sample) obtained, a full statistical analysis is carried out.

The reliability and accuracy of the results are highly dependent upon the number of realisations that are processed [8], which need to be sufficiently high to allow for the convergence of the results. Hence the efficiency and powerfullness of this method consists in keeping to a minimum the number of realisations needed so that the size of the experiment can be reduced.

This aspect impacts mainly on the definition of the random parameter. However a reasonable level of computational efficiency can be achieved if an appropriate sampling method is used to randomly generate the input parameter sets. In this work a Variance reduction technique (i.e Latin Hypercube sampling method) [2,3] is adopted to reduce the variance of the output population without disturbing the population expected value and keeping to a minimum the number of realisations.
Moreover the reliability of results is proven by means of the evaluation of Monte-Carlo experiment convergence: analyses at increasing number of realisations are performed and the convergence of both the probability of exceeding a threshold and the coefficient of variation for the output are kept under control.

As an alternative a second approach is developed: a preliminary part (point i to iii) is defined based on the DOE approach, while the Montecarlo Simulation technique is implemented to obtain output sample:

i. A full factorial plane is suitably design to account for parameter interaction and non linearity. [9], the factors level are representative of the parameters range of variation;
ii. Non linear simulations are performed for the combinations at point i), and the output (i.e. vehicle critical speed) for each combination is computed
iii. The output of step ii) are used to defined a polynomial relationship between output deviation and the parameter variation. The validity of the relationship is then properly verified
iv. Random parameter values are generated for the inputs according to their assumed probabilistic distribution;
v. For each realisation of the random variables set defined at point iv) the corresponding output is computed on the basis of the relationship assessed at point iii)
vi. For the output sample obtained a full statistical analysis is then carried out

The reliability and accuracy of the results provided by this approach is highly dependent on the definition of the relationship between output variability and input uncertainty. To correctly represent this connection, a 3-level full factorial plane is design as to account for non linearity and interactions. In the designed plane, three values are selected for each uncertain input, corresponding to the extremes and central point of the input range of variation.

Considering all different combinations in the values of the parameters under study, a total of $3^n$ “experiments” can be defined, being $n_p$ the number of parameters being varied.

5 Results: effect of parameter uncertainty in the model of vehicle component

In this section the propagation of uncertainty from component performance to critical speed is analysed for the yaw damper case according to the methodologies described in section 3 and 4.

The main hypothesis here is that uncertainty is associated to modelling, hence the same value of stiffness $k_d$ and damping $c_d$ is attributed to all vehicle yaw dampers. Moreover during the analyses the vehicle first bogie appeared, for the considered train, to be more critical towards critical speed then the rear one, thus all the analyses are focused on the behaviour of the front bogie.

In the following, first the simplest method is applied allowing for the smallest number of simulations, then the MCS approach is implemented and compared to the DOE&MCS one on the basis of 100 input samples.

5.1 OAT analysis

According to the OAT approach, a single parameter variation is simulated and the correspondent critical speed as well as the sensitivities coefficient are computed from partial derivatives calculation.

Table 5.1 reports the critical speed value for different combination of the damper damping $c_d$ and stiffness $k_d$ quantities.

| Stiff. [kN/m] | M | 9625 | 11323 | 13022 | 11323 | 11323 |
| Damp. [kNs/m] | 345 | 345 | 345 | 293 | 397 |
| Vehicle Critical speed [km/h] | 264 | 271 | 278 | 266 | 274 |
| Sensitivity coefficient | 0.004 [(km/h)/kN/m] | 0.080 [(km/h)/kNs/m] |
| Std Critical speed [km/h] | 3,87 |
| 0,15% | 260 |
| 99,85% | 283 |
| Pf(Vcr<264 km/h) | 0.035 |

Table 5.1 : OAT results for yaw damper modelling uncertainty

Furthermore the linear relationship assumed between input uncertainty and output variability allows to perform a simplified statistical investigation (i.e mean and standard deviation value or extreme values)
of the propagation of uncertainty as summarised in table 5.1. Besides if a limit on the critical speed is defined, a probability of failure can be assessed for the railway vehicle. In this study the threshold for the probability calculation is chosen in accordance to vehicle performance and corresponds to the maximum speed required by EN 14363 during acceptance tests in tangent track. Although through this simplified analysis it is possible to obtain some information on the propagation of uncertainty with minor simulation effort, it is not of immediate understanding which parameter most affects the variation of critical speed.

5.2 Montecarlo Simulation techniques analysis

For a more complete evaluation, the Monte Carlo simulation (MCS) technique is implemented. By means of LHS method, 100 samples representing random parameter values for the stiffness $k_d$ and damping $c_d$ are generated according to their probabilistic distribution (section 2) (see figure 5.1).

![Histograms](image)

**Figure 5.1:** Distribution of random variable generated according to LHS method

For each realisation a non linear simulation of running dynamics in tangent track is carried out so as to define the matching critical speed according to the methodology described in section 2. Finally a statistical analysis of the obtained output sample is performed in terms of:

i. Verification of Normal Probability distribution or Probability distribution identification

ii. Mean value

iii. Standard deviation

iv. 99.85 and 0.15 percentile

v. Probability of exceeding a threshold

Figures 5.2 a,b show the verification of probability distribution for critical speed sample computed. The paper plot [2] in figure 5.2a proves a good agreement between sample probability density function and the theoretical normal distribution as far as small variations around the mean value occur, while discrepancies can be found in the tails of the distribution. Contemporarely the histogram (figure 5.2b) for the critical speed sample clearly shows that a normal distribution tends to be conservative on the high tail, but less conservative if the low tail is analysed. Additionally the number of samples considered appears not enough to cover all the output variability, in fact they are located mainly around the mean distribution value, whereas just few of them can be found on the tails.
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In table 5.2 are reported the main outputs of the statistical analysis: the mean value of the distribution is found to be almost the critical speed for the vehicle in nominal condition, while the probability of failure shows that for the considered case, the threshold is approximately the 0.15% of the distribution itself.

<table>
<thead>
<tr>
<th>Mean Critical speed [km/h]</th>
<th>270.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std Critical speed [km/h]</td>
<td>2.22</td>
</tr>
<tr>
<td>0.15% percentile [km/h]</td>
<td>263.7</td>
</tr>
<tr>
<td>99.85% percentile [km/h]</td>
<td>276.9</td>
</tr>
<tr>
<td>Pf (Vcr&lt;264 km/h) [km/h]</td>
<td>0.0016</td>
</tr>
</tbody>
</table>

Table 5.2: MCS results for yaw damper modelling uncertainty

Alternatively to the MCS method, the combination of DOE and MCS approach is applied. Based on the DOE theory a full factorial plane is defined and the relationship between the critical speed and the varied parameters identified.

For the analysed case, since the same value of series stiffness \( k_d \) and damping \( c_d \) is attributed to the vehicle dampers, the factors to be varied are 2, \( n_p =2 \), hence the total number of numerical experiments to be performed is 9 (section 4).

For all these experiments, the vehicle critical speed is computed (section 2) and the dependence of this quantity on damper parameters is then approximated over the entire range of Maxwell model stiffness/damping values using a polynomial expression (eq. 4.6) of the type [9]

\[
\hat{V} = V_0 + a_1 \cdot x_1 + a_2 \cdot x_2 + a_3 \cdot x_1 \cdot x_2 + a_4 \cdot x_1^2 + a_5 \cdot x_2^2 + a_6 \cdot x_1 \cdot x_2^2 + a_7 \cdot x_1^2 \cdot x_2 + a_8 \cdot x_1^2 \cdot x_2^2
\]  

(4)

Where \( a_i \) are the coefficients of the relationship model, calculated applying the least square method to the data, while \( x_1 \) corresponds to the variation of Maxwell model stiffness \( k_d \) and \( x_2 \) to the variation of the Maxwell model damping \( c_d \).

Such a relationship is able to take into account the non-linearities as well as the interaction effects between parameters. The accuracy of the implemented relationship is verified carrying out the analysis of the residuals (i.e. residual mean value and distribution (figure 5.3 a and b)) and comparing an approximated value (\( \hat{V} \)) provided by the polynomial interpolation with the outcome of a numerical simulation (V) as reported in table 5.3.
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Table 5.3: Verification of polynomial relationship reliability

<table>
<thead>
<tr>
<th>Maxwell model parameters</th>
<th>Stiffness [kN/m]</th>
<th>Damping [kNzs/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated critical speed</td>
<td>10894</td>
<td>374</td>
</tr>
<tr>
<td>Simulated critical speed</td>
<td>270.63</td>
<td></td>
</tr>
<tr>
<td>Differences in %</td>
<td>270.39</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.08</td>
<td></td>
</tr>
</tbody>
</table>

For a given value of serial stiffness and damping the critical speed is computed both applying the polynomial relationship and the simulation. Table 5.3 shows that only slight differences can be found in the calculation of the critical speed. The relationship is demonstrated to be sufficiently accurate for the modelisation of critical speed dependence on yaw damper parameter variability.

Once defined the relationship, it is possible to apply the classical MonteCarlo simulation techniques using the same sample analysed previously with the MCS. Then for each realisation the corresponding critical speed is computed on the basis of the model assessed (eq. 4). Finally the same statistical analysis showed in figure 5.2 and table 5.2 is carried out and the results compared.

Figures 5.4 a and b summarise the differences between the statistical quantities (i.e mean values, coefficient of variation CV (\(\sigma/\mu\)) and percentile) obtained applying the two complex methodologies, demonstrating that the two methods are almost equivalent in terms of results. However the combination of DOE and MCS allows for a 80% reduction of simulation time.

In both cases the Normal distribution appeared to be not suitable for the representation of output sample distribution, however this result is strongly influenced by the number of realisation considered. In fact for the case under study the tails of the output distribution(figure 5.2b) are weakly represented by the considered realisations.
Figure 5.4: MCS and DOE&MCS comparison: a) mean value and 0.15% and 99.85% comparison b) CV comparison

Using the DOE & MCS approach, which is the fastest one, the minimum number of simulations required for the convergence of the MCS is computed in order to correctly analyse the propagation of uncertainty. Figures 5.5a and b show the trend with sample size of the two parameters [8] used for the verification of convergence: both the probability of exceeding a threshold and the critical speed coefficient of variation ($\sigma/\mu$) become constant in correspondence of 1000 realisations.

Figure 5.5: Convergence of MCS: a) Probability of failure convergence b) Coefficient of variation convergence

According to the performed analysis, the effect of uncertainty in the model of yaw damper can be properly studied at least with a 1000 samples. Clearly the application of a MCS approach it is not feasible due to the effort in terms of simulation time (i.e more then three days), hence then DOE & MCS method is implemented and the statistical quantities describing the critical speed sample computed as reported in figure 5.6. Increasing the sample size allows for a better distribution of sample, as demonstrated by the histogram in figure 5.6b. Furthermore the Gaussian distribution hypothesis appears to be valid also in correspondence of the tails.
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Besides the DOE & Monte-Carlo method allows for further information concerning the propagation of uncertainty: defining importance factors derived from Quadratic combination method [7]. These quantities provide a non-dimensional quantification of the effect that the input parameters have on the critical speed in dependence of the region where the variation occurs. As an example figure 5.7 shows the values of the importance factors in the case of variation around the mean value of the parameter population (variable nominal value), clearly the damper stiffness play a key role in the propagation of uncertainty. This kind of information is extremely useful when dealing with vehicle certification because provides information concerning the impact that different components have on the vehicle performance.

5.3 Simplified and complete model comparison

The results of the OAT approach and the DOE&MCS are compared in table 5.4, in terms of resulting mean value, standard deviation as well as 0.15 percentile and 99.85 percentile. A non-negligible difference is observed on the standard deviation, this bears implications on the tails of the two distributions. If the mean values are analysed it appears that they correspond almost the critical speed the vehicle has when in nominal configuration. Furthermore due to the higher standard deviation value, the probability of the critical speed exceeding the imposed thresholds has similar value, besides the OAT evaluation is conservative, being higher the probability of failure computed. It can be then concluded that for the case study considered here, the simpler method provides results which are enough conservatives to perform a probabilistic assessment of the vehicle critical speed.
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<table>
<thead>
<tr>
<th></th>
<th>OAT</th>
<th>DOE&amp;MCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Critical speed [km/h]</td>
<td>271.0</td>
<td>270.8</td>
</tr>
<tr>
<td>Std Critical speed [km/h]</td>
<td>3.9</td>
<td>2.8</td>
</tr>
<tr>
<td>0.15% percentile [km/h]</td>
<td>260.0</td>
<td>262.7</td>
</tr>
<tr>
<td>99.85% percentile [km/h]</td>
<td>283.0</td>
<td>280.0</td>
</tr>
<tr>
<td>Pf (V&lt;264 km/h) [km/h]</td>
<td>0.035</td>
<td>0.0058</td>
</tr>
</tbody>
</table>

Table 5.4: OAT and DOE&MCS method comparison

6. Conclusions

The paper presented a study, jointly carried out by SNCF and Politecnico di Milano, finalised to the investigation on how parametric uncertainty can be treated in the framework of virtual homologation of railway vehicles in respect to vehicle dynamics, considering uncertainty in the yaw damper properties and considering full correlation between parameters of all dampers in the vehicle. This type of uncertainty can be associated to inaccuracy in the modelling of a vehicle component which, at least to some extent, can be represented in the mathematical model of the vehicle as a deviation of model parameters from their 'true' value.

Three approaches with different level of complexity were proposed in the paper. They are completely new in the railway field, because entirely based on statistical methods.

The analysis of the propagation of uncertainty from the parameters input in the vehicle mathematical model to the results of running dynamics, the achieved results demonstrated that by means of numerical methodology it is possible not only to estimate a railway vehicle critical speed including parameter uncertainty effects in a simple and cost-effective way, taking advantage from the use of appropriate numerical tools, but also to provide information concerning the impact that different components have on the vehicle performance.

Further development of the work proposed here might consist in evaluating uncorrelated uncertainty on the yaw damper parameters as well as uncertainty on other important components which strongly influence vehicle running dynamics.

8. References


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