Estimation of Braked weight percentage for Electro-dynamic brake

Ilija Tanackov¹, Dr Dragomir Mandic², Dr Djordje Kopic³

1. Notation and units

\[ BP_{Pn+Ed} \] - braked weight percentage of co-action of pneumatic and electro-dynamic braking system (%)
\[ BP_{Ed} \] - pneumatic braked weight percentage (%)
\[ BP_{Pn} \] - electro-dynamic braked weight percentage expressed in train braked weight percentage (%)
\[ F_{edb} \] - electro-dynamic brake braking force (kN)
\[ I_p \] - electro-dynamic (A) excitation current
\[ I_r \] - electro-dynamic brake (A) rotor current
\[ v_s \] - speed of equalization of the excitation current braking power and rotor current (km/h)
\[ v_{ba} \] - initial speed of braking application (km/h)
\[ L_b \] - train braking distance achieved by application of pneumatic braking rigging in metres (m)
\[ L_{Ed}^{+} \] - train braking distance achieved by co-action of pneumatic and electro-dynamic brake in metres (m)
\[ \Delta L_b = L_b - L_{Ed}^{+} \] - difference of previous braking distances in metres (m)
\[ BW_{Pn} \] - braked weight of pneumatic brakes (t)
\[ W \] - train weight (t)

2. INTRODUCTION

The braked weight percentage is a measure of train brakes efficiency [1]. The calculation of braked weight percentage for different braking regimes is specified in UIC leaflet 544’1 [2]. The braking tables are a basis for practical application of railway brakes. The parameters specified in the tables are: the speed of brake application, the braked weight percentage, gradient of the railway line on which the braking is applied and the basic braking product, the braking distance. Under practical conditions the braking distance depends on the train composition, speed, braked weight, terrain configuration, status of air tanks, brake type and many other factors [3].

Electro-dynamic locomotive brake [4, 6, 12] is characterized by exceptional economies, flexibility and reliability. However, the electro-dynamic brake efficiency at low train speeds makes impossible a full estimate of its braking coefficient. Therefore an electro-dynamic brake is always a supplementary brake to pneumatic brakes. With co-action of electro-dynamic and pneumatic brakes, a higher efficiency of pneumatic brakes causes a lower participation of electro-dynamic brake in braking action and vice versa, which means that electro-dynamic brake has a variable efficiency, that is a variable braking percentage.

A question arises whether the variable braked weight percentage of the electro-dynamic brake can be expressed in braked weight percentages of pneumatic brakes. The following pages of this paper outline a mathematical procedure for forming the analytical form by means of which the electro-dynamic braked weight percentage is calculated.

3. BASIC CHARACTERISTICS OF ELECTRO-DYNAMIC BRAKE APPLICATION

With a constant value of route gradient, the braking distance is directly proportional to initial speed of braking application, and indirectly proportional to train braked weight. The co-action of pneumatic and

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electro-dynamic brakes gives a shorter braking distance than a sole application of pneumatic brakes. Hence, for the same speeds of braking application, the use of electro-dynamic brake leads to a growing of train braked weight percentage. There are two basic reasons for a growing of train braked weight percentage by the application of electro-dynamic brake.

Co-action of two braking systems: The pneumatic brake is a regular equipment of railway vehicles and gives the basic of the train braked weight percentage. Electro-dynamic brake is a supplementary brake. It is known from conditions in practice that non-equation (1) is applicable here:

\[ BP_{Pn+Ed} \geq BP_{Pn} \]  \hspace{1cm} (1)

The other reason arises from the fact that the co-action of pneumatic and electro-dynamic brakes has a higher efficiency than the sums of individual efficiencies of braking systems [5, 7]. Expressed in symbols, through of braked weights percentages, under practical conditions, the non-equation (2) applies:

\[ BP_{Pn+Ed} \geq BP_{Pn} + BP_{Ed} \]  \hspace{1cm} (2)

The electro-dynamic brake increases the power of pneumatic brakes: This increase is not high. For brakes with cast iron brake shoes, with pressure in brake cylinders of 3.8 bar and “G” braking regime, the maximum power increase of pneumatic brakes is 1.4%. The reason lies in mutual thermic unloading and a faster introduction in lower speed zones in which both pneumatic brake with cast iron shoes and electro-dynamic brake have higher efficiency. Here the increase of pneumatic brake power leads to an increase in nominal of the train braked weight percentage. This analysis first of all indicates a notable complexity of the combination of two braking systems.

4. THE SIMULATION MODEL DESCRIPTION

For the simulation research requirements of coordinated operation of electro-dynamic brake of class 441 locomotive and pneumatic brakes with brake shoes in the Yugoslav Railways freight wagons, a simulation model has been developed. The model dynamic basis is based on unvariable system of material points [7, 8]. The simulation model has the following abilities:
1. Testing of train movement on a route of arbitrary geometrical characteristics and categories of the line static loading (the maximum downgrades are up to 100 per thousand, minimum curve radius 300 m),
2. Forming of an arbitrary freight train composition with up to 40 wagons, where each wagon is separately loaded, introducing for each wagon the data on operating status of brake rigging, the position of brake handle and age of brakes. When the train is formed, the percentages of braked weight percentage of wagons and train are calculated [7, 12],
3. The arbitrary application of tractive force on the outer contour of class 441 locomotive traction log ,
4. A full support to a complex braking logic by simulating Oerlikon FV4A brake valve operation. The choice of braking mode (“P” or “G”), through development of pressure in train pipe line and brake cylinders with the value of pressurerupture speed of 250 m/sec), the possibility of brake release, operation on two distribution pressures, at prescribed changenover speeds (at 55 km/h) etc. A full description is given of the connection “brake device – train pipe – brake cylinder – brake rigging – brake shoe – wheel tyre”. Friction coefficients correspond to brakes with brake shoes, depending on the material used (made of cast iron or composite JURID BK 64),
5. The applied braking current of electro-dynamic brake in the model is \( I_p = I_s = I = 960 A \). The most recent research in the field of modeling electro-dynamic brake of class 441 locomotive (graphic 1) resulted in a general formula suitable for application in simulation models (formula 3):
The computer program of the model is prepared in Microsoft Visual Basic. It contains 9 forms: headline, four forms for data input, one for simulation and three for results presentation. The output results are the total performance and mean performance per meter of movement and the tractive forces power, braking power of pneumatic and electro-dynamic brakes, powers of permanent and temporary resistances. The output results contain also the figure on mean speed, acceleration and adhesion coefficients, necessary for implementation of braking simulation without wheel blocking. All mentioned values also have a possibility of graphical review (the updating is made per each meter run during the simulation). Figure 1 presents the screenshot forms for forming the train, figure 2 shows screenshot forms for riding simulation, and figure 3 presents screenshot forms for graphical and numerical analyses of the simulation process.
5. PREPARATION OF INPUT DATA

The data consist of five blocks with 48 braking simulations each. Trains of the following composition were tested:

1. Locomotive 441 and 20 Eas wagons. The train braked weight is 525 tons. Five trains were tested: train weighing 1312 tons with 40% of braked weight percentage, train weighing 1050 tons with 50% of braked weight percentage, train weighing 875 tons with 60% of braked weight percentage, train weighing 750 tons with 70% of braked weight percentage and train weighing 656 tons with 80% of braked weight percentage.

2. 441 locomotive and 30 Gas wagons. The train braked weight is 825 tons. Five trains were tested: train weighing 2062 tons with 40% of braked weight percentage, train weighing 1650 tons with 50% of braked weight percentage, train weighing 1375 tons with 60% of braked weight percentage, train weighing 1178 tons with 70 of braked weight percentage and train weighing 1031 tons with 80% of braked weight percentage.

3. 441 locomotive and 40 Fals wagons. The train braked weight is 1085 tons. Five trains were tested: train weighing 2712 tons with 40% of braked weight percentage, train weighing 2170 tons with 50% of braked weight percentage, train weighing 1808 tons with 60% braked weight percentage, train weighing 1550 tons with 40% of braked weight percentage and train weighing 1356 tons with 80% of braked weight percentage.
The initial speeds of braking application are 30, 40, 50, 60, 70, 80, 90 and 100 km/h. The brakings till stopping are accomplished with and without applying the electro-dynamic brake. During the braking also the locomotive pneumatic brake was applied. The results obtained of the achieved braking distances are presented in tables 1–5.

### Table 1: Simulation results for trains with 40% braked weight percentage

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6. INTERPRETATION OF RESULTS

For the purpose of estimating the electro-dynamic brake braked weight, graphics 2, 3 and 4 present the braking distances of trains with different braked weight percentages, with different initial braking speeds by electro-dynamic brake (marked with +Ed) and without electro-dynamic brake (marked with –Ed) (data from tables 1–5).

For train with 20 Eas wagons with 40% braked weight percentage and electro-dynamic brake (40+Ed), the braking distances are between the values for trains with 50% and 60% braked weight percentage, respectively, without electro-dynamic brake application. Hence, for such conditions the braked weight percentage of electro-dynamic brake is significantly above 10% of the braked weight percentage of pneumatic brake. The braking distances of a train with 50% of braked weight percentage and with electro-dynamic brake (50+Ed) almost completely matches with the braking distance of a train with 70% of braked weight percentage without applying the electro-dynamic brake (70-Ed), which means that the braked weight percentage obtained by action of electro-dynamic brake is somewhat below 20%. For train with 60% of braked weight percentage and with the applied electro-dynamic brake (60 +Ed), we can conclude that the braked weight percentage of electro-dynamic brake is about 20% because it matches with train braking distances of train with 80% of braked weight percentage without electro-dynamic brake (80-Ed) (graphic 2).

Analysis on train with 30 Gas wagons and 40% of braked weight percentage, indicates that the braked weight percentage of electro-dynamic brake is somewhat below 10%. The braking distances of train with 50% of braked weight percentage, with electro-dynamic brake (50+Ed) matches with the braking distance of train with 70% of braked weight percentage, without electro-dynamic brake (70-Ed), which means that electro-dynamic brake in this case has about 10% of braked weight percentage. On the same train with 60% of braked weight, the braked weight percentage of electro-dynamic brake is somewhat higher than 10% of braking weight percentage (Graphic 3).
As in the case of trains with 20 Eas and 30 Gas wagons, a similar analysis can be made on a train with 40 Fals wagons. On trains with 40% and 50% of braked weight percentage, the braked weight percentage of electro-dynamic brake is notably below 10%, while on a train with 60% of braked weight percentage, the electro-dynamic brake braked weight percentage is about 10% (Graphic 4).

If we carry out the analysis of trains of the same composition, we can conclude that with an increased braked distances, i.e. by reducing the total train weight, a percentage of braked weight of electro-dynamic brake increases. If we carry out an analysis for different composition trains which differ significantly by weight, but have the same braked weight percentage, we can see that on the lighter train the the braked weight percentage of electro-dynamic brake is higher. E.g. a train with 20 Eas wagons with the same braked weight percentages has about twice as low weight, than the train with 40 Fals wagons (tables 1 through 5). But, electro-dynamic brake on train with 20 Eas wagons has twice as big braked weight percentage for the same percentage of braked weights of trains!

The explanation of the results obtained for the same composition trains, but with different braking percentages and for trains of different compositions, but the same braking percentages, points to the fact that electro-dynamic brake has a clearly expressed braked weight, but its distribution along the overal weight of the train, can give a percentage, in a standard way, of the train braked weight. Because of that the braked weight of electro-dynamic brake distributed on heavier trains gives a lower braked weight percentage. It also should be noted that in cases presented in graphics 2, 3 and 4, the estimated braked weight percentage has a decreasing trend, i.e. it is approaching to lower percentages of braked weights of pneumatic brakes. The
reason for this occurrence is probably a consequence of low braking weight of electro-dynamic brake in higher speed zones (small area of performance integral on $F,v$ diagram at higher speeds).

### 7. ANALYTICAL EXPRESSION OF BRAKED WEIGHT PERCENTAGE OF ELECTRO-DYNAMIC BRAKE

For initial speeds of braking application which are as high or in excess of 60 km/h, the shortening of braking distance occurred by the application of electro-dynamic brake and braked weight of the train can be calculated by means of expression (4). By applying the theory of correlation it has been established that the values of shortening the braking distances (data from tables 1 through 5) and values of shortening obtained on the basis of formula (4) have a correlation coefficient of 0.99247.

$$\Delta L_b = \frac{31000 + 1800 \cdot v_{bs}}{BW_{pn}} - \frac{27000 + 1400 \cdot v_{bs}}{W} \quad (4)$$

Hence, expressed implicitly, the shortening of braking distance by the application of electro-dynamic brake is in function of the initial speed of braking application and the braked weight percentage equals:

$$\Delta L_b = L_b(v_{bs}, BP_{pn}) - L_b^{Ed}(v_{bs}, BP_{pn} + BP_{Ed}) = \frac{31000 + 1800 \cdot v_{bs}}{BW_{pn}} - \frac{27000 + 1400 \cdot v_{bs}}{W} \quad (5)$$

The functional dependence of simulated braking distances on the braked weight percentage and the vehicle speed is given explicitly by expression (6). By applying the correlation theory it has been established that the values of simulated braking distances (data from tables 1 through 5) and braking distances obtained from expression (6) have a correlation coefficient of 0.99463.

$$L_b = 332 \cdot e^{0.01901 \cdot v_{bs} - 0.0143 \cdot BP_{pn}} \quad (6)$$

By substitution of expression (6) for expression (5) we obtain expression (7):

$$332 \cdot (e^{0.01901 \cdot v_{bs} - 0.0143 \cdot BP_{pn}} - e^{0.01901 \cdot v_{bs} - 0.0143 \cdot (BP_{pn} + BP_{Ed})}) = \frac{31000 + 1800 \cdot v_{bs}}{BW_{pn}} - \frac{27000 + 1400 \cdot v_{bs}}{W} \quad (7)$$

By rearrangement of expression (7) as per $BP_{Ed}$, we obtain expression (8) for calculation of braked weight percentage of electro-dynamic brake for a given speed of brake application (in excess of 60 km/h) and given train braked weight percentage.

$$BP_{Ed} = -69.93 \cdot ln \left[ 1 - \frac{31000 + 1800 \cdot v_{bs} - 27000 + 1400 \cdot v_{bs}}{332 \cdot e^{0.01901 \cdot v_{bs} - 0.0143 \cdot BP_{pn}}} \right] \quad (in \ braked \ weight\%) \quad (8)$$

Since: $BP_{pn} = 100 \cdot \frac{BW_{pn}}{W} \quad (9)$

---

*In the presented case, owing to the application of cast iron braking shoes, the braking is applied through zones of high and low braking. Below 55 km/h, due to switchover speed, the overlapping of planes in figure 2 takes place. For speed below 55 km/h other formula (3) parameters are adequate, which, due to the lack of space are not outlined in the paper. For brakes with brake shoes made of composite materials and disc brakes, the procedure for calculation of braked weight percentage of electro-dynamic brake is made continuously, it is simpler, and it is made without division into several cases.*
We conclude that the percentage of braking weight of electro-dynamic brake \((I_p = I_s = I = 960A)\) depends on the train braked weight, train weight and the initial braking speed, i.e. this is presented by expression (10), the percentage of braked weight of electrodynamic brake is:

\[
BP_{Ed} = -69.93 \cdot \ln \left(1 - \frac{31000 + 1800 \cdot \nu_{hs} - 27000 + 1400 \cdot \nu_{hs}}{BW_{pn} \cdot 332 \cdot e^{-0.01901 \cdot \nu_{hs} - 1.43 \frac{BW_{pn}}{W}}}ight)
\]

The results of application of formula (10) for simulated trains, are given in table 6. The graphical presentation of results for train of braked weight of 525 tons is shown in figure 4. For train with braked weight of 825 tons and 1085 tons, respectively, we can conclude by looking at the table that graphical presentations are similar, providing that the values are lower than presented in figure 4.

Table 6: Percentage of electro-dynamic brake braked weight

<table>
<thead>
<tr>
<th>Speed km/h</th>
<th>Train braked weight percentages (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(BP_{pn} = 40)</td>
</tr>
<tr>
<td>v=60</td>
<td>25.69</td>
</tr>
<tr>
<td></td>
<td>15.22</td>
</tr>
<tr>
<td></td>
<td>11.26</td>
</tr>
<tr>
<td>v=70</td>
<td>23.71</td>
</tr>
<tr>
<td></td>
<td>10.47</td>
</tr>
<tr>
<td>v=80</td>
<td>21.56</td>
</tr>
<tr>
<td></td>
<td>12.93</td>
</tr>
<tr>
<td></td>
<td>9.60</td>
</tr>
<tr>
<td></td>
<td>11.70</td>
</tr>
<tr>
<td></td>
<td>8.71</td>
</tr>
<tr>
<td>v=100</td>
<td>17.29</td>
</tr>
<tr>
<td></td>
<td>10.50</td>
</tr>
<tr>
<td></td>
<td>7.83</td>
</tr>
</tbody>
</table>

- Train with braked weight of 525 t
- Train with braked weight of 825 t
- Train with braked weight of 1085 t
8. CONCLUSION

The mathematical procedure for estimation of braked weight percentage of electro-dynamic brake, presented in this paper, has dealt with a small set of conditions where the braking is applied on railways: only one operating characteristic of one electro-dynamic brake (960 A, locomotive 441 YR), pneumatic brakes were tested only in zones of high braking rate, brake shoes are made of cast iron, working pressure in brake cylinders is 3.8 bar, the braking regime is “P”, and the braking is applied on a straight line and horizontally. The presented results nevertheless lead to some conclusions:

1. With a growing speed the braked weight percentage of electro-dynamic brake decreases,
2. With increased weight the braked weight percentage of electro-dynamic brake decreases,
3. The highest braked weight percentage is achieved by electro-dynamic brake on trains the percentage of braked weights of which is between 50 and 60% (on the train with cast iron shoes).

It is assumed that a general formula can be found which, beside the outlined variables (speed, braked weight and train weight) could introduce also the general characteristics of electro-dynamic brake (according to formula 3), so that also a functional connection can be established between operating currents of electro-dynamic brake. Also, the function should incorporate the initial fall of pressure in train pipe. All this should be separately tested also for brakes with linings made of composite materials and for disc brakes.

The electro-dynamic brake braked weight percentage during the train ride on a downgrade would significantly express an impact of electro-dynamic brake, so that the down-grade magnitude can be introduced among the potential variables for estimation of braked weight percentage of electro-dynamic brake under specific conditions [11]. The mathematical procedure presented can be applied also in estimation of efficiency of dynamic and non-adhesion brakes such as aero-dynamic [4, 9] or rail brake with eddy currents.

The basic pre-requisite for correct derivation of the presented mathematical procedure is to achieve a high correlation coefficient between the collected data and analytical expressions describing the distribution of such data. The application of the presented procedure requires a high number of data. They can be collected through experiments on a real model, but it certainly would be more purposeful to do this with a simulation model which has been validated by railway administrations [10].

The most appropriate for observation by the presented mathematical procedure are passenger train sets with the installed electro-dynamic brake. They have only small alterations in the total weight and a constant
braked weight, which significantly simplifies a process of finding the estimated expression for the braked weight percentage. On the other hand, the driving of sets is abundant with brakings which especially justifies a frequent application of electro-dynamic brake, which has proven to be cheaper than the application of pneumatic brakes.

Literature: