Measurement of Tractive Force in the Creep Region
and Maximum Adhesion Control of High Speed Railway Systems

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Abstract

The acceleration and deceleration rate of the train depend on the tractive force. When the wheels of the train slip on the rails, the torque is decreased to avoid the continuous slipping. This reason is that the tractive coefficient between the wheels and the rails has a peak at a certain slip velocity. But this adhesive phenomenon is not clearly examined or analyzed. Thus we have developed a new adhesion test equipment. In this paper, we measured the tractive force with this equipment, and clarified the adhesive phenomenon. Then we proposed a new tractive force control and verified the effectiveness of the proposed control scheme.

1. Introduction

The strong demands for Shinkansen are the speed up of the trains, and the increase of the passenger transportation capacity while preserving the safety of high-speed railway systems. One possible approach for this is to enhance the speed adjustment capability. The end results are 1) the safety will be improved by reducing the distance for the urgent stop, 2) the train running density can be improved by increasing the average speed for the existing lines with many curves, that is called the high-density-scheduling. Generally, the speed adjustment performance can be enhanced through increasing driving or braking force that is mostly achieved by the maximum adhesion force between wheels and rails.

It is said that the tractive force between the wheel and the rail depends on the slip velocity between the wheel and the rail. And it is also said that this tractive force increases when the slip velocity increases a little, but if the slip becomes too large, it now goes down. However, it seems that there is no elaborated paper in which this phenomena were actually measured under the repeatedly measurable and reproducible conditions [1]~[3]. In [2], [4], the measured data on the peak of the tractive coefficients have large variations.

On the other hand, there are a number of papers on the re-adhesion control. In [4], under the assumption that the tractive force has a peak at a certain slip velocity, the slipping condition was estimated from the slip velocity, and the torque was reduced. In [5], the motor speed reference is reduced for re-adhesion. In [6], this approach was improved, and the torque characteristic of the motor has the drooping. Also in [7], the slip reference speed is adjusted based on the error of the actual slip and the slip reference. In [8], the slipping was detected from the slip speed of the multiple axles, and the tractive force was estimated. As a result, a sophisticated anti-slip re-adhesion
control was proposed and tested in the actual trains. In [5]–[8], the slipping was estimated from the slip velocity. However in [9]–[12], the tractive force was directly estimated by the disturbance observer, and the avoidance of the slipping and the use of the maximum adhesion force were proposed. In [9], [10], from the division of the derivative of the estimated tractive force based on the observer and the derivative of the slip velocity, the derivative of the tractive coefficient was calculated, and the maximum tractive force control was proposed. In [11], [12], also from the derivative of the estimated tractive force based on the disturbance observer, the anti-slip control was proposed. In [9]–[12], only the simulation results were presented.

In this paper, the following will be described. (1) The precise measurement method for the tractive force and the tractive coefficient are proposed, and these values are repeatedly measured by the newly developed adhesion testing equipment. Through the measurements, it is confirmed that the tractive coefficient has a peak at the certain slip velocity. (2) Based on the proposed real time tractive force measurement method, a new maximum tractive force control is proposed, and the proposed approach was verified through the test equipment. (3) Future problems on the proposed approach are summarized, and the road map for the further developments for this approach is clarified.

The chapter 1 is the introduction of the paper, and in the chapter 2 a new measurement method of the tractive coefficient is proposed, and using the newly developed adhesion test equipment the measured data are shown. In the chapter 3 a new maximum tractive force control is proposed and verified by the test equipment.

The chapter 4 is on the discussions and the chapter 5 concludes this paper.

2. Proposal of the Tractive Coefficient Measurement

2.1 Mathematical Model for Tractive Force Measurement

For the measurement of the tractive force, one axle model as shown in Fig. 1 is assumed, and the following differential equations are proposed [9].

\[
J \frac{d\omega}{dt} = T_v - Fr - B\omega \quad (1)
\]
\[ M \frac{dv}{dt} = F - C v^2 \quad (2) \]

\[ F = \mu Mg \quad (3) \]

where \( T_r \) is the input motor torque, \( \omega \) is the motor speed, \( v \) is the train speed, \( F \) is the tractive force, \( \mu \) is the traction coefficient, and \( g \) is the gravity acceleration. Other parameters are shown at Table 1.

Table 1. Parameters for simulations.

<table>
<thead>
<tr>
<th>J</th>
<th>R</th>
<th>B</th>
<th>M</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel inertia</td>
<td>Wheel radius</td>
<td>Viscosity friction</td>
<td>Train mass</td>
<td>Aerodynamic coefficient</td>
</tr>
<tr>
<td>37.6</td>
<td>0.5</td>
<td>0.32</td>
<td>20000</td>
<td>0.63</td>
</tr>
<tr>
<td>(kg m²)</td>
<td>(m)</td>
<td>(kg m/sec)</td>
<td>(kg)</td>
<td>(sec²/m³)</td>
</tr>
</tbody>
</table>

The slip velocity is defined as the absolute value of the difference between the train body velocity and the wheel velocity, as follows.

\[ \Delta \omega = \omega_r - \nu \quad (4) \]

The following state equation is obtained if the tractive force \( F \) changes as a step function.

\[
\begin{bmatrix}
\dot{\omega} \\
\dot{T_r}
\end{bmatrix}
= \begin{bmatrix}
-\frac{B}{J} & -\frac{1}{J} \\
0 & -\frac{1}{T_r}
\end{bmatrix}
\begin{bmatrix}
\omega \\
T_r
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{J}
\end{bmatrix}
T_e
\quad (5)
\]

where the tractive torque \( T_r^\Delta = Fr \quad (6) \)

If the wheel angular velocity is measurable, the following output equation is obtained.

\[ \omega = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \omega_r \\ T_e \end{bmatrix} \quad (7) \]

Equation (5) and (7) are observable, and the state variables can be observed. Thus, the minimal order observer is derived as follows [13].

\[ \hat{T}_r = \frac{\omega_c}{s + \omega_c} [T_e - (J \omega + B_n) \omega] \quad (8) \]

where the suffix of \( n \) denotes the nominal value. Also the \( \omega_c \) can be free parameter and it is the pole of the observer. The above-mentioned algorithm was reported in [9],[10] by authors; however, the experimental measurement was not done yet.

### 2.2 The Adhesion Test Equipment and Parameters

#### 2.2.1 The Adhesion Test Equipment

The adhesion test equipment was developed for clarification of the adhesion phenomena and also for the precise measurement of the tractive force. The size of the setup is one eight of the actual Shinkansen wheel. The outlook is shown in Fig.2 and the function block diagram is in Fig.3. The material of the setup is the same as the Shinkansen wheel and rail. The diameter of the train wheel (train wheel for abbreviation) is one eight of the actual train wheel. And the rail is represented five times larger diameter wheel (rail wheel for abbreviation) as shown in Fig.4. The detailed parameters
are listed in Table 2. The 1:8 reduction gear is installed between the rail wheel and rail side driving motor. The axle weight is continuously controllable by the servomotor.

Fig. 2: Adhesion test equipment.

Fig. 3: Function block of the adhesion test equipment.
In this paper, all data are based on the crude data in this adhesion test equipment. For example, the speed 10 km/h of the train wheel is effectively 8 times larger in the actual model, that is 80 km/h. In this setup the maximum speed is designed to be 40km/h or the 8 times larger equivalent train speed of 240 km/h.

### 2.2.2 Parameter Measurements

For the estimation of the tractive force $\hat{T}_L$ in (8), the inertia $J_o$, the viscosity friction constant $B$ and the observer pole $\omega_c$ are required. The $\omega_c$ is selected by the trial and error approach in the experiments, and the other two parameters are selected as follows.

Table 2. Parameters for experiment equipment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>3106(mm)</td>
</tr>
<tr>
<td>Width</td>
<td>107.5(mm)</td>
</tr>
<tr>
<td>Height</td>
<td>1115(mm)</td>
</tr>
<tr>
<td>Total weight</td>
<td>2000(kg)</td>
</tr>
<tr>
<td>Train side motor</td>
<td>7.5(kw)</td>
</tr>
<tr>
<td>Rail side motor</td>
<td>7.5(kw)</td>
</tr>
<tr>
<td>Rated torque</td>
<td>47.7(Nm)</td>
</tr>
<tr>
<td>Train side maximum revolution</td>
<td>1600(rpm)</td>
</tr>
<tr>
<td>Rail side maximum revolution</td>
<td>2500(rpm)</td>
</tr>
<tr>
<td>Train side wheel diameter</td>
<td>107.5(mm)</td>
</tr>
<tr>
<td>Rail side wheel diameter</td>
<td>537.5(mm)</td>
</tr>
<tr>
<td>Maximum load</td>
<td>686(N)</td>
</tr>
</tbody>
</table>
As for the $B\omega$, first the train wheel was rotated until the motor temperature becomes the constant without contacting the rail wheel. Second, the speed reference is increased from zero to the high speed. Under the steady state, the speed and the torque reference of the inverter were measured at several speed points. As a result, the $B=0.07$ $\text{N}\cdot\text{m}\cdot\text{sec}/\text{rad}$ was obtained.

As for the $J$, the following procedure was employed. In equation (1), if $F=0$ or without contacting the rail wheel, and the train wheel is accelerated or decelerated at the constant rate, the speed increase $\omega_{\Delta}$ and speed decrease $-\omega_{\Delta}$ can be expressed in the next equations.

$$\Delta \omega = \frac{1}{J} \int_{t_0}^{t_1} (T_e - B\omega) \, dt$$

$$-\Delta \omega = \frac{1}{J} \int_{t_1}^{t_2} (T_e - B\omega) \, dt$$

where $[t_0, t_1]$ is the acceleration period and $[t_1, t_2]$ is the deceleration period. By controlling the inverter it is easy to make $t_1 - t_0 = t_2 - t_1$ and the velocity change rate is the same. The subtracting equation (10) form (9) yields the next equation.

$$2\Delta \omega = \frac{1}{J} \int_{t_0}^{t_1} T_e \, dt - \int_{t_1}^{t_2} T_e \, dt$$

Under the assumption that the torque reference of the inverter is the same as the actual torque, the inertia was obtained to be $J=0.021$ $\text{kg}\cdot\text{m}^2$ from equation (11).

### 2.3 Measurement of the Tractive Force and the Tractive Coefficient

From the steady state that the both wheels are rotating at the same surface velocity under the contact, the torque of the motor of the train wheel was suddenly increased, and the slipping was generated. The tractive force was measured by the disturbance observer in (8). The tractive coefficient is calculated by the division of the estimated tractive force and the axle weight based on the definition in (3). The $\omega_c$ is 100 $\text{rad}/\text{sec}$.

The results are shown in Fig.5 ~ Fig.7. Fig.5 depicts the tractive coefficient and the slip ratio, Fig.6 and Fig.7 denotes the motor torque and train wheel speed, in which the symbols (+ × ∗ ☐) show the lapse of time. If we repeat this experiment, the wheel surface is terribly damaged by the instantaneous heat, thus we also accelerate the rail wheel by the following equation, which means that the mass $M$ of the train is accelerated by the estimated tractive force.

$$\omega_{\text{rail}} = \frac{1}{r^2} \int \frac{T_l}{\text{weight}} \, dt$$

On the other hand, the slip ratio in the horizontal axis in Fig.5 is calculated from (4), however, in this figure the calculated slip ratio is the output of the low pass filter of the cut-off frequency $\omega_c$, which is the same as the observer pole. This reason is that since the estimated tractive force is obtained through the low pass filter, both tractive force and slip velocity should have the same time delay. Fig.5 implies that the maximum tractive coefficient is about 0.28 around the slip ratio of 0.05. Above this slip ratio the
tractive efficient gradually decreases. This tendency was observed through various experiments with different axle weight and different train speed.

Fig. 5: Tractive coefficient (axle weight 500 N).

Fig. 6: Motor torque.
2.4 Discussion of the Measurement

To enhance the accuracy of the measurement of the tractive force, the following points should be more elaborated.

1. The precise measurement of the viscosity friction $B\omega$
2. Measurement under the constant speed of the rail wheel
3. Additional measurement of the tray loss such as wind loss of the wheels
4. Check the influence of different axle weight
5. Check the influence of the steady state speed of the rail wheel just when the torque is suddenly increased
6. Large variation of the tractive coefficient by the water wet surface condition

As for (1), the measured $B\omega$ is shown in Fig.8, in which the $B$ is not constant. At present the experimental data were taken under the almost constant $B$ region, however, the $B$ should be on line changed to have more precise tractive coefficient. As for (2), the surface of the iron wheels was worn out in this condition, thus we quitted this setting. We measure the (3), but it was very small and negligible. As for (4) and (5) we had the very similar curve like Fig.5. We will investigate more on (4), (5) and (6).
3. Maximum Tractive Force Control

3.1 Proposed Algorithm

The tractive coefficient has a peak in terms of the slip velocity as shown in Fig.5, thus the maximum tractive force can be available at the point when \( \frac{dF}{ds\text{lip}} = 0 \).

The following approximation is assumed.

\[
\frac{dF}{ds\text{lip}} \approx \frac{d\hat{F}}{dt} \frac{1}{dt}
\]  

(13)

To calculate the \( \frac{d\hat{F}}{dt} \), the quasi-derivative \( \frac{-sg}{s + g}, g = 100 \) of the estimated tractive force is proposed [15]. The value of g is heuristically selected so as to reduce the effect of the noise. In [9], [10], the derivative of the tractive force is modeled as a part of the state equation, and this value is estimated by the full order observer, however, it is sensitive to the noise around the peak of the tractive force. Also \( \frac{ds\text{lip}}{dt} \) is directly calculated from the velocity of the train and rail wheels, but as mentioned in section 2.3, it is low pass filtered with the cut off frequency of \( \omega_\text{c} \), and quasi-derivative is taken.

The optimal slip velocity which makes the tractive force maximum is generated in the slip reference generation block as shown in Fig.9.
Fig. 9: Block diagram of maximum tractive force control.

The optimal procedure is based on the steepest gradient method as follows.

1. \(\frac{d\hat{F}}{dslip}\) is calculated from \(\frac{d\hat{F}}{dt}\) and \(\frac{dslip}{dt}\).

2. The slip velocity reference is \(slip_{ref}(k+1) = slip_{ref}(k) + \alpha \frac{d\hat{F}}{dslip}\), where \(\alpha\) is a constant, and in the experiment it was selected as \(1.0 \times 10^{-5}\).

3. go back to (1)

Dividing the sum of the train speed and slip velocity by the radius of the train wheel generates the motor speed reference. The motor speed regulation is done by the PI controller. The features of this proposed algorithm are (1) tractive force is directly calculated by the disturbance observer, (2) the optimal slip velocity is generated as the reference, (3) steepest gradient method is used for optimization.

3.2 Verification by Experiments

The proposed algorithm was tested using the adhesion test equipment as mentioned in Chapter 2. In the experiment the train speed is needed, thus using (2), the train speed was calculated by the computer in real time. The train mass is selected as the axle weight divided by the gravity constant \(g\). The axle weight was changed in several cases, however similar curves were observed, thus one of the typical available tractive coefficient with the axle weight 400(N) is shown in Fig.10.
The initial slip reference was given as 1 km/h and the proposed maximum tractive control was begun at \( t=0 \) (s). It is observed that the most of the tractive coefficients were located around the peak region. In this experiment a pulse sensor of 1024 (ppr) was used, however, in the existing train system usually 60 (ppr) sensor is widely installed. In the reference [17], the simulation studies of 60 (ppr) sensor was performed and the very good results were obtained in the mid and high speed region. Due to the hardware modification in the experiments, the final results with 60 (ppr) sensor will be shown at the conference. From Fig.10 it is confirmed that the maximum tractive force is actually realized in the experiments based on the proposed algorithm.

3.3 Discussions on the Maximum Tractive Force Control

We believe that there are following items that we should investigate more.

(1) In the very low speed region, the pulse from the pulse sensor is very little, thus the train speed estimation is difficult, which, in turns, makes the tractive force estimation difficult.

(2) If the tractive force changes very much, the effectiveness of the propose algorithm is not yet checked.

As for (1) we are investigating in [16], and if less than one pulse is observed within one sampling interval, the instantaneous speed observer looks very attractive. As for (2) we will make experiments, however, in simulations the validity of the proposed algorithm was checked in [17].
4. Discussions

The future problems for the measurement and control were already mentioned in section 2.4 and 3.3. As we reviewed the literatures in the introduction, the direct estimation of the tractive force is relatively new approach in the train field, however, in the field of traction control of electric vehicles, it was already reported to be measurable [18]. In case of the rubber tire, the slip ratio becomes larger when the tractive coefficient has a peak compared with iron wheels. And it was reported that it was observed that the tractive force had a peak at a certain slip speed. Very important information for the direct measurement of the tractive force is the exact values of parameters, and also the train velocity. The following future problems should be investigated.

(1) How to measure the precise values of inertia $J$ and viscosity friction $B$ and the train speed $v$. Or new approach to estimate the slip velocity without the train speed [19].
(2) Train speed estimation at the very low speed region
(3) Theoretical equation for the tractive force
(4) Tested data at the actual Shinkansen trains

Authors are investigating the above targets, however, due to the limitation of the available space those will be reported at the next opportunity.

5. Conclusions

First, the precise measurement method for the tractive force is proposed, and these values are repeatedly and re-productively measured by the newly developed adhesion testing equipment. Through the measurements, it is confirmed that the tractive coefficient has a peak at the certain slip velocity. Second, based on the proposed real time tractive force measurement method, a new maximum tractive force control is proposed, and the proposed approach was verified through the testing equipment. Third, future problems on the proposed approach are summarized, and the road map for the further developments for this approach is clarified.

References