Experimental determination of the acoustic source impedance of the air conditioning duct system

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Summary

The acoustic wave from the acoustic sources or flow noises, which generated in the ducts of the air conditioning system, moves not only through the tested ducts into the environments, but also on the other direction (to source). Depending upon the source impedance of the air conditioning system the acoustic wave reflection in the ducts is completely different; the reflected acoustic wave is overlaid with the acoustic wave moving into the environments and leads to a measuring error. One obtains the best result of measurement, if no reflected acoustic wave from the source of the air conditioning system exists. So it is very important to determine the source impedance of the air conditioning system before the actual measurements. But numerous publications concerning the source impedance determinations show that this are very complex and complicated.[2,3] For this reason here a simple measuring method was developed, in order to reduce the expenditure and the measuring time.

Introduction

The internal impedance of the acoustic source becomes after the equation (1) infinitely, if the diaphragm of the acoustic driver consists of sound-hard material and the length of the connecting piece for acoustic sound driver mounting $I \rightarrow 0$ is. Then the acoustic wave moves from the source on the one hand only outward by the inspection item which can be measured, on the other hand toward test stand.
With one sound-hard locked pipe is the acoustic tube impedance:

\[ Z = -j Y_0 \cot(k \rho) \]  

(1)

Figure 2.10: Schematic representation of source-impedance for the test facility and its analogous electrical circuit \([4,5]\)

The acoustic relation between the microphone positions is 1 and 2 in matrix way of writing:

\[
\begin{bmatrix}
    p_1 \\
    v_1
\end{bmatrix} = \begin{bmatrix}
    A_{12} & B_{12} \\
    C_{12} & D_{12}
\end{bmatrix} \begin{bmatrix}
    p_2 \\
    v_2
\end{bmatrix}
\]

or

\[
\begin{bmatrix}
    p_2 \\
    v_2
\end{bmatrix} = \begin{bmatrix}
    D_{12} & -B_{12} \\
    -C_{12} & A_{12}
\end{bmatrix} \begin{bmatrix}
    p_1 \\
    v_1
\end{bmatrix}
\]

(2)
In the equation (2) is the sound pressure in level 2:

\[ p_2 = D_{1,2} \cdot p_1 - B_{1,2} \cdot v_1 \]  

(4)

The equation (4) is divided by the sound pressure \( p_1 \) and supplies:

\[ \frac{p_2}{p_1} = D_{1,2} - \frac{B_{1,2}}{Z_1} \]  

(5)

In the equation (5) acoustic impedance \( Z_1 \) resolved to [13] and the sound pressure by the transfer functions \( H_{21} \) is replaced:

\[ Z_1 = \frac{B_{1,2}}{D_{1,2} - \frac{p_2}{p_1}} = \frac{B_{1,2}}{D_{1,2} - H_{12}} \]  

(6)

Here the impedance for the following frequencies \( f \) is because of standing wave not valid [1]:

\[ f = \frac{ma_0}{2l_1^2}(1 - Ma^2), \quad m=1, 2, 3 \ldots \]  

(7)

That means the acoustic equations:

\[
\begin{bmatrix} p_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} A_{0,1} & B_{0,1} \\ C_{0,1} & D_{0,1} \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix} 
\]  

(8)

and

\[
\begin{bmatrix} p_E \\ v_E \end{bmatrix} = \begin{bmatrix} A_{E,0} & B_{E,0} \\ C_{E,0} & D_{E,0} \end{bmatrix} \begin{bmatrix} p_0 \\ v_0 \end{bmatrix} 
\]  

(9)

The equations (8) into the equations (9) inserts and results:

\[
\begin{bmatrix} p_E \\ v_E \end{bmatrix} = \begin{bmatrix} A_{E,0} & B_{E,0} \\ C_{E,0} & D_{E,0} \end{bmatrix} \begin{bmatrix} A_{0,1} & B_{0,1} \\ C_{0,1} & D_{0,1} \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix} 
\]  

(10)

If the tube cross-section area between the distance \( l_1 \) and \( l_E \) is alike, these equation becomes simpler:
\[
\begin{bmatrix}
    p_E \\
    v_E
\end{bmatrix}
= \begin{bmatrix}
    A_{E,l} & B_{E,l} \\
    C_{E,l} & D_{E,l}
\end{bmatrix}
\begin{bmatrix}
    p_l \\
    v_l
\end{bmatrix}
\]

In accordance with equation (11) is the acoustic source-impedance of the test stand in level E:

\[
Z_E = \frac{A_{E,l} \cdot Z_l + B_{E,l}}{C_{E,l} \cdot Z_l + D_{E,l}}
\]  

(12)

and here

\[A = \cos(k_0 l), \quad B = j \cdot Y_0 \cdot \sin(k_0 l)\]

\[C = j \cdot Y_0 \cdot \sin(k_0 l), \quad D = \cos(k_0 l)\]

\[Y_0 = \frac{a_0}{S}; \text{ characteristic Impedance, } l: \text{ tube length}\]

**Experimental set up**

The test facility consists essentially of:

- a pipe with two microphone connector (B&k type 4133, 1/2"
- a t-shaped acoustic driver(speaker)
- a chamber (not necessarily)
- a calibration pipe

Here a high-grade steel pipe with an inside diameter \(D = 47\) mm and a wall thickness of 1.5 mm were used, which ensures a high wall firmness to keep on the structure borne small to the microphone. The connecting piece of the t-pipe should be as short as possible, so the acoustic driver diaphragm about on the same level of the internal tube wall. In order to adjust sensitivity and the phase difference between the pair of microphones, the microphones before the actual measurement were calibrated. By the acoustic driver is the white noise generated; as signal source is a signal generator of the company B&k used. Depending upon the interesting frequency range the correct speaker must be used. In the laboratory measurement the used Midrange(type Selenium D=250) has indicated with very deep frequencies (below 100 Hz) and very high frequencies (above 4 kHz) lack of transfer. This lack can be reduced by use of the correct type of loudspeaker (woofer or tweeter) for each the interesting frequency range.
Conclusion

The "ideal test facility" is most favorable, which does not produce a sound reflection like an infinitely long pipe. With an infinitely long pipe acoustic tubing impedance is $Y_0$ (characteristic impedance), since in the entire frequency range no reflecting wave occurs here. During the measurement the muffler, which is intended for the actual measurement, was attached to the supply pipe. The expansion chamber length was changed by the adjustable internal disk, whereby the acoustic impedance of the inspection item is modified. The analyzed results of measurement in figure 1 show a very good agreement with different expansion chamber lengths and an almost ideal test stand, since the acoustic source impedance of the test facility corresponds to characteristic tube impedance. It occurs indeed that the acoustic wave is because of the long pipe length up to the pressure air tank and the upstream muffler hardly reflected. In the area of the frequencies 1150, to 2300 and 3450 Hz is the impedance mismatching, otherwise almost perfect agreement with the theory. The invalid frequencies are in the equation (13) for the microphone distance of $l_2 = 0.15$ m and an ambient temperature of $T_0 = 20.7\, ^\circ C$ without flow:

$$f = \frac{m \cdot a_0}{2l_2} \left(1 - Ma^2\right) = 1146, 2292, 3438 Hz, \ldots, \quad m = 1, 2, 3, \ldots \quad (13)$$

![Figure 1: Test result of source impedance for the air conditioning duct test facility](image)
Bibliography


