Evaluation Method for Vibration-Reduction Efficiency of Floating Slab Track

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Abstract

The floating slab track is commonly used to reduce the vibration and noise induced by running train. However, the floating slab track is usually very expensive compared with normal ballast track. Therefore, to maximize the effect of vibration reduction, it is important to design the floating slab track considering the characteristics of running train, track, and building. In this paper, a numerical method for evaluating the efficiency of vibration reduction of floating slab track built on the floor of building on commercial business is developed. In the analyses, the train is modeled using the 4-masses consisting of the car body, the bogie, and the two wheel-sets connected with springs and dampers each other. The slab track and substructure are simulated using continuous beams supported by uniformly distributed springs and dampers. The coefficient of bottom springs and dampers supporting substructure are evaluated considering the dynamic behavior of the floor of substructure equivalently.

The compatibility conditions at the contact points between wheels and rails are used to derive the system equation of motion. The PSD functions of track irregularity are adopted as inputs for random analyses. The numerical analyses are carried out to investigate the effects of train speed, stiffness and damping of slab-pad, and track irregularity upon vibration reduction in substructure under the track.

To verify the appropriateness of the developed numerical method, experimental investigations are performed in the Bu-Chun subway station on the main subway line in Seoul. The results are compared with those obtained using the numerical model. The two results show good agreement to each other, at least in the trend of responses. The developed numerical method may be applied in the optimal design of floating slab track.
1. Introduction

While increase of train speed inevitably results in higher noise and vibration, there is more and more a growing tendency toward a stricter environment standard. One of the major requirements for railroad construction, therefore, is to ensure satisfactory noise-proofing and vibration-proofing to meet the standard. With a very high population density and resultant adjacency between railroads and residential areas, it is also in urgent need to develop more efficient and economical sound-proofing and vibration-proofing technology to countermeasure public grievances.

Thus, many countries have extensively employed vibration-reduction track as an efficient method to abate noise and vibration inside stations or neighboring buildings. Korea has also been making use of it more and more since that type of track was first laid inside Puch'on Station on Subway Line 1(RED). Without a precise design based on train(track interaction, however, there is a possibility that safety and passenger comfort will be put at risk. It is also possible that investment effect will diminish in spite of high construction cost unless the design maximizes vibration-reduction efficiency via dynamic analysis.

Aimed at helping optimal design of floating slab track, therefore, this study tried to develop a numerical method for evaluating the vibration level of the structure under floating slab track. Interaction analysis of train and track system was carried out after the equation of motion was derived from that of both train and each track system by applying compatibility condition at the contact points between wheels and rails. For this purpose, the track is modeled by continuous support beam system and the train by 3-mass system. The presented method was verified by comparing the result actually measured at the construction site with the one obtained from the analysis. The review was then carried out of the efficiency of vibration-proof of track according to the change of such parameters as train speed, spring stiffness and damping coefficient of floating slab track, and the track irregularity.

2. High Frequency Analysis of Track

2.1 Calculation of wheel and rail interaction force

When modeling the track on structures in the form of 4-layered continuous beam with a view to performing dynamic analysis, the wheel and rail interaction force at the front wheel \( F_{H1} \) and the one at the rear wheel \( F_{H2} \) can be given by Eq. (1)

\[
\begin{bmatrix}
F_{H1} \\
F_{H2}
\end{bmatrix} = \begin{bmatrix}
K_H (y_{w1} - y_{r1} - y_{s/w1}) \\
K_H (y_{w2} - y_{r2} - y_{s/w2})
\end{bmatrix} = \begin{bmatrix}
y_{w1} \\
y_{w2}
\end{bmatrix} - \begin{bmatrix}
A_{w1} \\
A_{w2}
\end{bmatrix}
\]

(1)
Here, $\bar{y}_{r1}$, $\bar{y}_{r2}$ and $\bar{y}_{w1}$, $\bar{y}_{w2}$ are the displacements of rail and wheel at their contact points, respectively, while $A_{w1}$, $A_{w2}$ are the respective admittances at the front wheel and the rear one, which are, in other words, the wheel displacement to unit harmonic force. $K_H$ is Hertzian spring stiffness at wheel and rail contact points, whereas $1/w_{r1}$, $1/w_{r2}$ are the surface roughness of wheel and rail at the front wheel and the rear one. From Eq. (1), the rail displacements become:

\[
\begin{bmatrix}
\bar{y}_{r1} \\
\bar{y}_{r2}
\end{bmatrix}
= \begin{bmatrix}
\bar{y}_{r/w1} \\
\bar{y}_{r/w2}
\end{bmatrix}
- \begin{bmatrix}
\left(\frac{1}{K_H} + A_{w1}\right)F_H1 \\
\left(\frac{1}{K_H} + A_{w2}\right)F_H2
\end{bmatrix}
\]  (2)

A formula is derived from the definition of rail admittance and expressed as follows

\[
\begin{bmatrix}
A_{r11} & A_{r12} \\
A_{r21} & A_{r22}
\end{bmatrix}
\begin{bmatrix}
F_{H1} \\
F_{H2}
\end{bmatrix}
= \begin{bmatrix}
\bar{y}_{r1} \\
\bar{y}_{r2}
\end{bmatrix}
\]  (3)

From Eq. (2) and Eq. (3), the correlation between interaction force and rail/wheel relative roughness can be expressed :
Using Eq. (3) and Eq. (4), the correlation between wheel/rail roughness and rail displacement can be derived:

\[ \ddot{y}_r = H_r(\dot{u})y_{wW} \]  

(5)

in which:

\[ \ddot{y}_r = \begin{bmatrix} \ddot{y}_{r1} \\ \ddot{y}_{r2} \end{bmatrix} \]  

(6)

\( H_r(\omega) \) is the transfer function indicating the rail response to unit wheel/rail roughness and is expressed as:

\[ H_r(\omega) = \begin{bmatrix} A_{r11} & A_{r12} \\ A_{r21} & A_{r22} \end{bmatrix} \begin{bmatrix} \frac{1}{K_H} + A_{w1} + A_{r11} \\ \frac{1}{K_H} + A_{w2} + A_{r22} \end{bmatrix}^{-1} \]  

(7)

The dynamic behavior of each track system can be obtained through (5) so long as wheel and rail admittances and wheel/rail relative roughness are provided.

2.2 The calculation of rail admittance

When modeling a ballasted track under moving harmonic excitation as 3-layered continuous beam by removing slab and slab-pad from the one in Figure 1, the equation of motion of substructure of track are given by:

\[ E_I \frac{d^4 y_1}{dx^4} + m_1 \frac{d^2 y_1}{dt^2} + k_1^*(y_1 - y_2) = F_h(t)\delta(x - vt)e^{i\omega t} \]  

(8-a)

\[ E_2 I_2 \frac{d^4 y_2}{dx^4} + m_2 \frac{d^2 y_2}{dt^2} + k_1^*(y_2 - y_1) + k_2^*(y_2 - y_3) = 0 \]  

(8-b)

\[ E_3 I_3 \frac{d^4 y_3}{dx^4} + m_3 \frac{d^2 y_3}{dt^2} + k_2^*(y_3 - y_2) + k_3^*y_3 = 0 \]  

(8-c)

Here, \( k_1^*, k_2^*, k_3^* \) are the vertical complex stiffness of rail pad, ballast, and foundation per unit length, respectively.

The response of the substructure of track to harmonic excitation can be expressed:
The partial differential to x and t is, in other words, the partial differential to relative coordinate \( r = x - vt \) and reads:

\[
\frac{\partial^n y}{\partial x^n} = \frac{\partial^n \tilde{y}}{\partial r^n} e^{i\omega t} , \quad \frac{\partial^n y}{\partial t^n} = (i\omega - v \frac{\partial}{\partial r})^n \tilde{y} e^{i\omega t} \quad (10)
\]

By substituting (10) for (8), the equation of motion can be expressed in the form of function only with relative coordinate \( r \). The response of the substructure of track can be obtained by performing Inverse Fourier Transform of \( \tilde{y}_1, \tilde{y}_2 \) and \( \tilde{y}_3 \), which are in turn the result of Fourier Transform of these formulae into s-domain. \( A_r (= \tilde{y}_r / F_H) \) can be acquired from the response to the harmonic excitation of substructure of track[ref].

### 2.3 The calculation of wheel admittance

When taking into consideration first and secondary suspension of rolling stocks and the mass of the bogie and the car body, the motion of rolling stocks due to the roughness can be modeled by 3-mass system as in Figure 2 and its equation of motion is:

\[(11-a) \quad M_i \ddot{y}_i + C_i (\dot{y}_i - \dot{\tilde{y}}_i) + K_i (y_i - \tilde{y}_i) = 0 \]

\[(11-b) \quad M_b \ddot{y}_b + C_s (\dot{y}_b - \dot{\tilde{y}}_b) + C_p (\dot{y}_b - \dot{\tilde{y}}_w) + K_s (y_b - y_i) + K_p (y_b - y_w) = 0 \]

\[(11-c) \quad M_u \ddot{y}_w + C_p (\dot{y}_w - \dot{\tilde{y}}_b) + K_p (y_w - y_b) = F_H \]

![Figure 2. 3-mass system](image-url)
Here, $M_t$ and $M_b$ are the mass of the car body and the bogie, respectively, while $M_w$ is unsprung mass including axles and wheels. $K_p$ and $K_s$ are spring coefficients of first and secondary suspension, whereas $C_p$ and $C_s$ are the damping coefficients of first and secondary suspension. The admittance of wheel, $A_w (= \frac{\ddot{y}_w}{F_p})$ can be obtained from $\ddot{y}_w(\omega)$, which is in turn the result of Fourier Transform of (11) into frequency domain.

### 2.4 Power spectrum density of wheel/rail roughness

At an arbitrary point $i$ on the surface of rail, PSD to roughness is reported to be inversely proportional to the cube of spatial frequency. Under this assumption, the PSD to roughness is expressed as:

$$S_{w_i}(\omega) = \frac{A}{(B + \Omega)^3} \quad (12)$$

Here, $\Omega = \frac{\omega}{v} = \frac{2\pi}{\lambda}$ means spatial frequency and $\omega$, $\lambda$ angular frequency, wave length, respectively. $A$ and $B$ are roughness constants. $A$ is the characteristic value responding to short wave length less than 3m and usually uses $1.0 \times 10^{-7}$, whereas $B$ is the one responding to long wave length and uses $0.36 \, \text{m}^{-1}$.

### 2.5 PSD response of the rail

Given $S_{w_i}(\omega)$ as track irregularity power spectrum, cross power spectrum density (CPSD) for two points on the same rail can be obtained from the concept of autocorrelation function. From the definition, $R_{r_{iw_i}}(\tau)$ for two points $i$ and $j$ becomes:

$$R_{r_{iw_i}}(\tau) = E[\ddot{y}_{r_{iw_i}}(t) \ddot{y}_{r_{iw_j}}(t+\tau)] \quad (13)$$

Taking into account $t_{ij} = (x_j - x_i)/v$, the difference in time between the arrivals of wheels to two points on the same rail, it becomes:

$$\ddot{y}_{r_{iw_i}}(t+\tau) = \ddot{y}_{r_{iw_i}}(t + \tau - t_{ij}) \quad (14)$$

Therefore, (13) is:

$$R_{r_{iw_i}}(\tau) = E[\ddot{y}_{r_{iw_i}}(t) \ddot{y}_{r_{iw_j}}(t + \tau - t_{ij})] = R_{r_{iw_i}}(\tau - t_{ij}) \quad (15)$$

CPSD for two points can be obtained from Wiener-Khintchine transform and reads:


\[ S_{r/w_0}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{r/w_0}(\tau) e^{-i\omega\tau} d\tau = S_{r/w_0}(\omega) e^{-i\omega t_0} \tag{16} \]

From (16), track irregularity spectrum matrix at two points becomes:

\[ S_{r/w}(\omega) = \begin{bmatrix} \frac{1}{e^{i\omega t_0}} & e^{-i\omega t_0} \\ e^{i\omega t_0} & 1 \end{bmatrix} S_{r/w_0}(\omega) \tag{17} \]

The rail spectrum response is then derived from (5) and (17) and reads:

\[ S_r(\omega) = \left[ H_r(\omega) \right] S_{r/w} \left[ H_r(\omega) \right]^* \tag{18} \]

Here, \( \left[ H_r(\omega) \right]^* \) is the complex conjugate of \( \left[ H_r(\omega) \right] \).

With PSD response of each track system acquired, 1/3 octave band acceleration level can be obtained from the integration of PSD between lower frequency \( f_l \) and upper frequency \( f_u \) and given as:

\[ dB(A_i) = 10 \log_{10} \left( \frac{\omega^4 S_{r_i}}{a_{ref}^2} (f_u - f_l) \right) = 10 \log_{10} \left( \frac{C \omega^k S_{r_i}}{a_{ref}^2} \right) \tag{19} \]

Here, the reference quantity of acceleration level in decibel, \( a_{ref} \) is \( 1.0 \times 10^{-5} \text{ m/sec}^2 \) and \( C \) is a constant, \( 2^{1/6} - 2^{-1/6} = 0.2316 \).

3. Modeling of substructure as equivalent beam supported by Winkler spring

Floating slab track is mainly laid to reduce the vibration of substructure under track. In order to evaluate it with the method presented in this study, it is necessary to reasonably model the substructure as an equivalent beam supported by Winkler spring. In general, it can be difficult to carry out simulation close to the real behavior by modeling 3D structure as 2D beam supported by Winkler spring. When taking into account only high frequency vibration of slab of the substructure, however, it can be properly modeled since the structural characteristics of the slab and its supporting condition have the biggest influence on the vibration of the slab.

Three parameters required for modeling the equivalent beam, i.e. the supporting spring stiffness \( k \), flexible rigidity \( EI \) and the mass per unit length \( m \) can be evaluated through the following three assumptions.

(1) The displacement due to the unit load exerted on the middle of slab whereon the track (shown in Figure 3) is laid is equal to the one due to the unit load on the equivalent model.
(2) One second of column spacing or the distance reducing the displacement into zero when load are exerted on the middle of slab is equal to the distance reducing displacement into zero in the equivalent model.

(3) Natural frequency of the slab of the substructure is equal to that of the equivalent model.

After acquiring central displacement ($u_0$), column spacing($L$), and natural frequency($\omega_n$) through the structural analysis of the substructure, the following formulae are derived from the above mentioned assumptions.

From assumption (1), $u_0 = 2 \sqrt{\frac{2EI}{k}} \frac{k}{L}$

from assumption (2), $\frac{3\pi/4}{L/2} = \frac{4}{\sqrt{k/4EI}}$

and from assumption (3) $\omega_n = \sqrt{k/m}$

From these three equations, $k$, $EI$ and $m$ can be evaluated.

Figure 3. Modeling of substructure as equivalent beam supported by Winkler spring
4. Verification of the analysis method through vibration measurement

In order to verify the presented method, comparison was made between the vibration level actually measured at the floating slab track in Puch'on Station with the one obtained from the analysis. Figure 4 shows both traverse and longitudinal cross section of the substructure with floating slab track.

![Image of cross sections](image)

(a) Traverse cross section

(b) Longitudinal cross section

Figure 4. Transverse and longitudinal cross section of the substructure with floating slab track.

Measurement matrix of vibration is shown in Figure 5 and the results measured at each track part and the slab of structure during train operation are shown in Figure 6.

![Image of measurement system](image)

Figure 5. Measurement system
Numerical analyses were carried out to evaluate vibration acceleration levels of floating slab track and substructure with due regard to local conditions of the measuring site. The properties and cross section used in the analyses are presented in Table 1 and in Figure 7, respectively.

Table 1. Property of Analysis Model
<table>
<thead>
<tr>
<th>Component</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail</td>
<td>Modulus of Elasticity</td>
<td>$2.06 \times 10^8$ (kN/m²)</td>
</tr>
<tr>
<td></td>
<td>Weight per Unit length</td>
<td>60.8 (kg/m)</td>
</tr>
<tr>
<td></td>
<td>Section Area</td>
<td>$7.75 \times 10^{-1}$ (m²)</td>
</tr>
<tr>
<td></td>
<td>Moment of Inertia of Section of Rail</td>
<td>$3.09 \times 10^{-5}$ (m⁴)</td>
</tr>
<tr>
<td>Rail Pad</td>
<td>Vertical Stiffness</td>
<td>$1.0 \times 10^5$ (kN/m)</td>
</tr>
<tr>
<td></td>
<td>Damping ratio</td>
<td>$\eta = 0.2$</td>
</tr>
<tr>
<td>Sleeper</td>
<td>Weight per Unit</td>
<td>244 (kg/Unit)</td>
</tr>
<tr>
<td></td>
<td>Interval Distance</td>
<td>0.6 (m)</td>
</tr>
<tr>
<td>Ballast</td>
<td>Weight per Unit Length</td>
<td>800 (kg/m)</td>
</tr>
<tr>
<td></td>
<td>Vertical Stiffness</td>
<td>$2.0 \times 10^5$ (kN/m)</td>
</tr>
<tr>
<td></td>
<td>Damping ratio</td>
<td>$\eta = 0.1$</td>
</tr>
<tr>
<td>Slab</td>
<td>Modulus of Elasticity</td>
<td>$3.0 \times 10^7$ (kN/m²)</td>
</tr>
<tr>
<td></td>
<td>Weight per Unit Length</td>
<td>2,200 (kg/m)</td>
</tr>
<tr>
<td></td>
<td>Section Area</td>
<td>0.88 (m²)</td>
</tr>
<tr>
<td></td>
<td>Moment of Inertia of Section of Rail</td>
<td>0.063 (m⁴)</td>
</tr>
<tr>
<td>Slab Pad</td>
<td>Weight per Unit</td>
<td>5,600 (kN/m)</td>
</tr>
<tr>
<td></td>
<td>Damping ratio</td>
<td>$\eta = 0.2$</td>
</tr>
<tr>
<td>Substructure</td>
<td>Modulus of Elasticity</td>
<td>$3.0 \times 10^7$ (kN/m²)</td>
</tr>
<tr>
<td></td>
<td>Slab Resonance Frequency</td>
<td>$f_n = 12.9$ Hz</td>
</tr>
<tr>
<td></td>
<td>Span Distance</td>
<td>25 m</td>
</tr>
<tr>
<td>Train</td>
<td>Train Running Speed</td>
<td>50 km/h</td>
</tr>
<tr>
<td></td>
<td>Unsprung Mass</td>
<td>1000.0 (kg)</td>
</tr>
<tr>
<td></td>
<td>Axle Distance</td>
<td>2.5 m</td>
</tr>
<tr>
<td></td>
<td>Hertzian Spring Contact Stiffness</td>
<td>$1.4 \times 10^6$ (kN/m)</td>
</tr>
<tr>
<td>Track Irregularity</td>
<td>PSD: $S(\omega)=A/(\Omega^2+B)^3$</td>
<td>$A = 1.0^{-7}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$B = 0.36$</td>
</tr>
</tbody>
</table>

One third octave band acceleration levels which are obtained from the analysis carried out on track with and without vibration-reduction system are shown in Figure 8. It is noted from the results that vibration level at substructure, in case of floating slab track, is significantly reduced even though there is no change in the vibration level at rail and sleeper. It is also known that floating slab track can reduce considerably structure-bone noise since it diminishes the vibration of substructure in higher frequency range.
Figure 8. 1/3 octave band acceleration levels of ballast track and floating slab system

Figure 9 shows the vibration level measured at both slab of floating track and substructure together with the analyzed one. It is recognized that the measured vibration level at the slab of structure is almost congruous with the analyzed one, whereas those at slab of track have some discrepancy. It is considered that the difference comes mainly from the simplified modeling of the complex vibration system. Another cause of the difference may also be induced by the fact that many parameters among the properties used in the analysis are not realistic but assumptive.

Figure 9. Vibration level of floating track and substructure

However, it is important to be able to evaluate the vibration of structure exactly in some degree since the main objective of floating slab track is to reduce the vibration of substructure. From this point of view, therefore, this method is considered to be useful in designing floating slab track since it can help to obtain considerably precise evaluation result of the effect of the dimensions and properties on the vibration of under structure.
5. Investigation of how the properties values of affect the vibration of substructure

The effects of individual parameters on the vibration of substructure are evaluated using the presented method. Each parameter of interest is, in turn, isolated and varied widely while the remaining parameters are kept at same values in Table 1. One third octave band velocity level of the substructure according to the passing train speed is shown in Figure 10-(a). The vibration reduction efficiency defined as the difference of overall velocity level between the substructure under ballasted track to the one under floating slab track is shown in Figure 10-(b). It is noted from the results that maximal effect occurs at a certain speed. From this, it can be deduced that the stiffness and damping coefficient of isolator need to be optimized for a fixed train speed when designing floating slab track. Figure 11 shows how the stiffness of isolator affects both vibration level and vibration reduction effect of substructure. Here, the ratio of stiffness change \( K/K_0 \) means the proportion of investigating stiffness of isolator, \( K \) to reference stiffness, \( K_0 \). The result attests that the lower the stiffness grows, the higher the vibration reduction effect becomes. Therefore, it is regarded desirable to minimize the stiffness as much as possible so long as it doesn't jeopardize the safety of track and train. From Figure 12 presenting vibration level and vibration reduction effect of substructure according to damping coefficients, it is acknowledged that though the increase of damping coefficient does raise reduction effect, its rate of increase diminishes with damping coefficients increasing. In order to review how track irregularity affects the vibration reduction effect of floating slab track, analysis of vibration level of substructure was carried out using different figures for A value of track irregularity PSD in (12). The analysis result in Figure 13 shows track irregularity does effect the increase of vibration level, but that it doesn't have any influence on vibration reduction effect. From this, it is noted that track irregularity doesn't have to be considered in designing floating slab track.
Figure 11. Analysis of vibration reduction effect according to the slab pad stiffness difference

(a) 1/3 Octave Band Vibration Level  
(b) Vibration Reduction Effect

Figure 12. Analysis of vibration reduction effect according to the pad damping ratio

(a) 1/3 Octave Band Vibration Level  
(b) Vibration Reduction Effect

Figure 13. Analysis of vibration reduction effect according to Track irregularity

(a) 1/3 Octave Band Vibration Level  
(b) Vibration Reduction Effect
6. Conclusion

This study developed a numerical method for evaluating the vibration reduction effect at under structure in order to get an optimized design of floating slab track. To verify its validity, comparison between the result of actual measurement and the analyzed one was carried out and it proved both results correspond to each other at a reasonable level. From the review of the vibration reduction efficiency of slab track according to the change of train speed, supporting stiffness, damping coefficient and track irregularity via the developed analysis method, the following were eventually obtained:

- Reduction of vibration of substructure under floating slab track is proportional to frequency. However, vibration of rail and sleeper doesn't show any change.
- The stiffness and damping coefficient of isolator need to be optimized for a fixed train speed when designing floating slab track.
- Vibration reduction is reversely proportional to slab-pad stiffness.
- Vibration reduction is proportional to damping coefficient but increase rate of reduction effect diminishes with damping coefficient increases.
- Though track irregularity increases vibration level, it doesn't affect vibration reduction effect.

7. Reference