Harmonic Analysis Model Based on 8-port Representation for Korean High Speed Railway

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Abstract - This paper proposes a new model for harmonic analysis in 2–25kV traction power supply system including inverted feeder, contact line, rails and auto-transformer. The system model is based on four-port representation which is an extension of two-port network theory.

In order to verify the proposed approach, we have analysed and tested real traction power feeding system focused on the amplification of harmonic current. The calculation results from the proposed approach and the measurement data from the test are widely described in the paper.

1. Introduction

Modern AC electric car has thyristor or PWM(Pulse Width Modulation)-controlled converters, which give rise to higher harmonics. The current harmonics injected from AC electric car is propagated through power feeding circuit. As the feeding circuit is a distributed constant circuit composed of RLC, the capacitance of the feeding circuit and the inductance on the side of power system cause a parallel resonance and a magnification of current harmonics at a specific frequency.

The magnified current harmonics usually brings about various problems. That is, the current harmonics makes interference in the adjacent lines of communications and the railway signalling system. Furthermore, in case it flows on the side of power system, not only overheating and vibration at the power capacitors but also wrong operation at the protective devices can occur.

Therefore, the exact assessment of the harmonic current flow must be undertaken at design and planning stage for the electric traction systems. From these point of view, this study presents an approach to model and to analyse traction power feeding system focused on the amplification of harmonic current.

The system model is based on 8-port representation which is an extension of 2-port network theory. 8-port representation for each component of the system has many advantages:

1) Overhead catenary lines which have mutual impedances as well as series impedances and shunt admittances can be easily represented by an equivalent 8-port model with appropriate circuit parameters.

2) Distributed circuit parameters for the line model can be defined. 8-port representation makes it possible to obtain the relationship the currents and the voltages for an auto-transformer with mid-point connected to the rails.

3) The entire system can be easily modeled by the combination of 8-port representation of each component in parallel and/or series.

4) Harmonic calculation can be performed straightforwardly.

The proposed algorithm is applied to a real AT(Auto-transformer)-fed system in which electric car with thyristor-controlled converters is running. In order to verify the proposed approach, we have analysed and tested real traction power feeding system focused on the amplification of harmonic current. The calculation results from the proposed approach and the measurement data from the test are widely described in the paper.

2. Modeling and Formulation

2.1 Harmonic Circuit Model for Traction Power Feeding System

The usual feeding systems of electrification are based on single phase 25 kV/50 kV. The feeding system is connected to three-phase power system to be supplied with large single-phase load.

AC feeding circuits supply vehicles with the power by 3 to 2 phase Scott transformer through feeder, contact wire and rail as shown in Fig 1. Auto- transformers are installed about every ten kilometers with circuit breakers which connect adjacent up and down tracks at sub-sectioning post(SSP). Substations (SS) are located every fifty kilometers and there is a sub-sectioning post(SP) midway between two substations. SP has circuit breakers which enable one feeding circuit to electrically separate from the other. They may be closed in case adjacent SS is out.
of service. Now let source impedance be $Z_{ss}$ and traction vehicle be harmonic current source on the side of phase M (or T).

2.2 Formulation by 8-port Representation
In a general way, it is desirable harmonic analysis should be performed on two-port representation for the sake of calculational convenience.
Harmonic current source is, however, connected to contact line and rail conductor on secondary sides of two ATs which of primary terminals are connected between contact line and inverted feeder in the circuit model as shown in Fig 1. What is more, there are capacitive admittances among three conductors (contact line, inverted feeder and rail conductor). They make it impossible to realistically model the traction power feeding system upon two-port representation.
For that reason, this paper proposes a new model for harmonic analysis in the traction power supply system including inverted feeder, contact line, rails and auto-transformer. The system model is based on four-port representation which is an extension of two-port network theory.

Four-port representation for each elements in the AC electrified railway system can be derived as follows:

1) The Modelling of a Auto-transformer
An auto-transformer at SP connected to the overhead catenary line in parallel may be modelled as shown in Fig 2.

![Fig 2. Equivalent circuit of Auto-transformer for SP](image)
The relationship between voltages and currents for AT at the end of the line (SP) can be derived as equation (1).

\[
\begin{bmatrix}
V_{n} \\
V_{r1} \\
V_{r2} \\
V_{r3} \\
I_{r1} \\
I_{r2} \\
I_{r3}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 2Z_{at} & 2Z_{at} \\
0 & 1 & 0 & 2Z_{at} & 2Z_{at} \\
0 & 0 & 1 & 2Z_{at} & 2Z_{at} \\
1 & 1 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
V_{1} \\
V_{2} \\
V_{3} \\
I_{1} \\
I_{2} \\
I_{3}
\end{bmatrix}
\]

(1)

The equivalent circuit of an auto-transformer at SSP is shown in Fig 3.

Fig 3. Equivalent circuit of Auto-transformer for SSP

On the basis of Fig 3, it is possible to obtain the relationship between voltages and currents for an auto-transformer at SSP.

\[
\begin{bmatrix}
V_{n} \\
V_{r1} \\
V_{r2} \\
V_{r3} \\
I_{r1} \\
I_{r2} \\
I_{r3}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
V_{1} \\
V_{2} \\
V_{3} \\
I_{1} \\
I_{2} \\
I_{3}
\end{bmatrix}
\]

(2)

2) Overhead Catenary lines

Overhead catenary lines have not only their self and mutual impedances but also shunt admittances. Equivalent T-type model for an overhead catenary line section can be represented with these parameters as shown in Fig 4.
Therefore, the relationship for voltage and currents can be rearranged as equation (5)

\[
\begin{bmatrix}
V_{T1} \\
V_{T2} \\
V_{T3} \\
V_{T4} \\
I_{T1} \\
I_{T2} \\
I_{T3} \\
I_{T4}
\end{bmatrix} = \begin{bmatrix} M_{V,2} & M_{I,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{V,2} & M_{I,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{V,2} & M_{I,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_{V,2} & M_{I,2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{V,2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{T1} \\
V_{T2} \\
V_{T3} \\
V_{T4} \\
I_{T1} \\
I_{T2} \\
I_{T3} \\
I_{T4}
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_{T1} \\
V_{T2} \\
V_{T3} \\
V_{T4} \\
I_{T1} \\
I_{T2} \\
I_{T3} \\
I_{T4}
\end{bmatrix} = \begin{bmatrix} M_{I,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{I,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{I,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{I,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{I,2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{I,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_{I,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{I,2} \end{bmatrix} \begin{bmatrix} V_{T1} \\
V_{T2} \\
V_{T3} \\
V_{T4} \\
I_{T1} \\
I_{T2} \\
I_{T3} \\
I_{T4}
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_{T1} \\
V_{T2} \\
V_{T3} \\
V_{T4} \\
I_{T1} \\
I_{T2} \\
I_{T3} \\
I_{T4}
\end{bmatrix} = \begin{bmatrix} M_{V,2} & M_{I,2} \end{bmatrix} \begin{bmatrix} V_{T1} \\
V_{T2} \\
V_{T3} \\
V_{T4} \\
I_{T1} \\
I_{T2} \\
I_{T3} \\
I_{T4}
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_{T1} \\
V_{T2} \\
V_{T3} \\
V_{T4} \\
I_{T1} \\
I_{T2} \\
I_{T3} \\
I_{T4}
\end{bmatrix} = \begin{bmatrix} M_{I,2} \end{bmatrix} \begin{bmatrix} V_{T1} \\
V_{T2} \\
V_{T3} \\
V_{T4} \\
I_{T1} \\
I_{T2} \\
I_{T3} \\
I_{T4}
\end{bmatrix}
\]

(5)

3) Harmonic Current Source

A harmonic current source in AC electrified railway system is, mainly, traction car, which can be considered as a harmonic current source injected from rail to contact line as shown Fig 5.
Fig 5. Harmonic Current Source
From this point of view, 8-port model for harmonic current due to traction car is represented as equation (6).

\[
\begin{bmatrix}
V_{F1} \\
V_{F2} \\
V_T \end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
I_{H1} \\
I_{F1} \\
I_{T1} \\
I_{F2}
\end{bmatrix} =
\begin{bmatrix}
V_{F1} \\
V_{F2} \\
V_T
\end{bmatrix} -
\begin{bmatrix}
I_{H1} \\
I_{F1} \\
I_{T1} \\
I_{F2}
\end{bmatrix}
\]

(6)

4) The Modelling of a substation
The equivalent circuit of phase M (or T) for a substation with an auto-transformer is shown in Fig 6 and Fig 7.
Where $Z_s$ implies the impedance of main transformer and power utility.

Fig 6. Equivalent Circuit Model for Power Utility and Main transformer at SS
Fig 7. Equivalent Circuit Model for Auto-transformer at SS

The 8-port model of equivalent circuit for the auto-transformer is represented as equation (7).

\[
\begin{bmatrix}
E_{u1} \\
E_{u2} \\
E_{u3} \\
E_{u4}
\end{bmatrix} =
\begin{bmatrix}
1 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
2 & 0 & 0 & 0 \\
0 & 0 & 2 & 0
\end{bmatrix}
\begin{bmatrix}
A & B & A & B \\
A & B & A & B \\
C & D & C & D \\
C & D & C & D
\end{bmatrix}
\begin{bmatrix}
V_{r1} \\
V_{r2} \\
V_{r3} \\
V_{r4}
\end{bmatrix}
\begin{bmatrix}
I_{r1} \\
I_{r2} \\
I_{r3} \\
I_{r4}
\end{bmatrix}
\]

(7)

Where,

\[A = \sqrt{2} Z_{u} \]
\[B = -\sqrt{2} Z_{u} \]
\[C = \sqrt{2} Z_{u} + Z_{AT} \]
\[D = -\sqrt{2} Z_{u} + Z_{AT} \]

Assuming \(V_s\) to be purely sinusoidal 60Hz voltage source, \(V_s\) become 0[V] for harmonic analysis.

3. Harmonic Analysis

Focused on the amplification of harmonic current, harmonic analysis for traction power feeding system will be described in this section. For analysis, 8-port model obtained in section 2 is used for harmonic analysis.

Now, the entire system can be easily modeled by the combination of Eight-port representation of each component in parallel and/or series. Therefore, following equation will result as follows.
The Voltage and current at SP can be obtained from equation (8). In like manner, the voltage and current at any point of the line can be calculated straightforwardly.

4. Case Studies
4.1 Input Data
A real traction power feeding system in reference [3] was used for the test system. The detail data can be found in the reference.

4.2 Results
In order to verify the proposed approach, we have analysed and tested real traction power feeding system focused on the amplification of harmonic current. The harmonic current through into SS is shown in Fig 8. From Fig 8, it is observed that the measured harmonic current is very close to the estimated result by the proposed approach.

Fig 8. Magnification of Current Harmonics

Fig 9. illustrates the impact of the catenary model on the result. The comparison have been performed between the results: 1 section ; 2 sections ; 3 sections ; more than 4 sections of Equivalent T-type model for an overhead catenary line. There are more differences in result as harmonic order is increased in case of 1 section and 2 sections of Equivalent T-type model. Therefore it is desirable to use more than 4 sections of Equivalent T-type model for an overhead catenary line. As a result, the resonance frequency is about 1500[Hz] and the magnification of harmonic current is about 6.5[pu].
Fig 9. The Impact of the Catenary Model on the Result

Fig 10. illustrates the harmonic current vs. location of vehicle. The resonance frequency is not depend on the location of vehicle as shown Fig 10. The magnification of harmonic is, however, a function of the position of a train. The farther is the train from the substation, the magnification of harmonic current is higher. These results are similar to those of reference [1] and give a basis that R-C bank should be installed at SP.

Fig 11. shows correlation between catenary length and harmonic Resonance. From the result, The resonance frequency is lower as catenary length is longer.

Fig 10. Magnification of Current Harmonics as a Function of the Position of a Train
5. Conclusions

This study presents an approach to model and to analyse traction power feeding system focused on the amplification of harmonic current. Through the research we can conclude the following:

1) It is desirable to use more than 4 sections of Equivalent T-type model for an overhead catenary line.
2) The resonance frequency is not depend on the location of vehicle. The magnification of harmonic is, however, a function of the position of a train.
3) The resonance frequency is lower as catenary length is longer.

References