Analyses of Contact Force Fluctuation between Catenary and Pantograph in a hanger span cycle.

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Abstract:
One of the most important subjects of overhead contact line and pantograph system is to improve the contact performance in high-speed operation. In the existing catenary equipment for Shinkansen lines, the contact force fluctuation in a support span cycle is comparatively small because a compound-type catenary is used and the whole tension is high. Therefore, the contact force fluctuation in a hanger span cycle is comparatively large. It has been confirmed in past researches that when the train speed is close to the wave velocity of contact wire, the contact loss ratio becomes extremely large and contact loss occurs in a hanger span cycle. It is necessary to clarify the causes of the contact force fluctuation between catenary and pantograph, and to propose an improvement method to reduce the contact force fluctuation. In this research, the relation between the wave motion of contact wire and the contact force fluctuation of pantograph is studied; the contact force fluctuation in a hanger span cycle is analyzed; and methods to reduce the contact force fluctuation are proposed.

The mechanism of contact force fluctuation in a hanger span cycle is considered as follows. That is, wave motion is generated by pantographs running under the dip of contact wire between hangers, reflects at the nearest hanger, becomes incident to the pantograph and causes contact force fluctuation. It is confirmed by theoretical analyses and fundamental experiments that the magnitude of the contact force fluctuation is proportional to the vertical velocity amplitude of the wave incident to the pantograph. The transfer coefficient of contact wire wave motion at hanger points is measured on a real overhead contact line by the wave separation measuring method and spectrum analysis method. The unevenness of contact wire is also measured on a real overhead contact line by the unevenness measuring method applied with the laser technique.

From the above study, we obtain the conclusions summarized below about the mechanism of contact force fluctuation and the improvement methods for high-speed operation.
1) It is shown that the contact force fluctuation of pantograph in high-speed operation is mainly caused by the incident wave of contact wire that is generated by the unevenness of contact wire and is reflected at hangers.
2) This influence becomes extremely large when the train speed is close to the wave velocity of contact wire. The amplitude of contact force fluctuation exceeds the stationary uplift force of a pantograph at the non-dimensional speed of about 0.7 or over.
3) Increasing the wave velocity of contact wire or decreasing the reflection factor of wave motion at hangers is effective to reduce the contact force fluctuation.

keywords:
Current collection, overhead contact line, contact wire, pantograph, wave motion
1. Introduction

For high-speed operation of electric railways, reducing the contact force fluctuation of pantograph is one of the most important subjects. It has been confirmed in past researches\(^1\) that when the train speed is close to the wave velocity of contact wire, the contact performance becomes extremely degraded and contact loss occurs in a hanger span cycle. It is necessary to clarify the causes of the contact force fluctuation between catenary and pantograph and propose an improvement method to reduce the contact force fluctuation.

In this paper, the relation between the wave motion of contact wire and the contact force fluctuation of pantograph is studied; the contact force fluctuation in a hanger span cycle is analyzed; and methods to reduce the contact force fluctuation are proposed.

2. Contact force fluctuation

The best contact condition is that the contact force between contact wire and pantograph is always constant and equal to the stationary contact force. But the contact force changes in high-speed operation due to several causes. The main causes are considered as (1) the fluctuation mechanism in a support span cycle, (2) fluctuation mechanism in a hanger span cycle, (3) unevenness of contact wire (undulating wear, for example), (4) aerodynamic disturbance and others. The magnitude of the contact force fluctuation in a support span cycle and that in a hanger span cycle are indices to evaluate the dynamic performance of overhead equipment.

Concerning the contact force fluctuation in a support span cycle, the following improvement methods have been shown in past researches\(^2\), (1) reducing the variability of catenary elasticity, and (2) increasing the whole tension of catenary. A heavy-compound catenary is normally used for Shinkansen lines in Japan. Fig.1 shows an outline of this catenary. Because this equipment consists of three wires (so-called “compound type”), the elasticity at the support point is comparatively small. The total tension of this catenary amounts to 53.9kN. Therefore, it is considered that the contact force fluctuation in a support span cycle is comparatively small, and that in a hanger span cycle is influential in high-speed operation.

![Diagram of Shinkansen catenary](image)

Fig.1 Standard overhead equipment for Shinkansen (Heavy-compound catenary)
3. Analysis of contact wire wave motion

3.1 Method of measuring wave motions

The wave propagating velocity (phase velocity) of the contact wire is one of the most important indices for the current collecting performance. The wave velocity is obtained by estimating the frequency at which the addition or subtraction of accelerations at two points is equal to 0. A measurement example is shown in Fig. 2. It is found that the wave velocity increases at high frequencies because of its flexural rigidity and that the measured values agree well with the calculation result.

In order to estimate the wave motion of the contact wire, it is necessary to separate the measured vibration of the contact wire into the forward propagating wave and the backward one. If the wave motion $y(x,t)$ is expressed by Eq. (1), two wave motions propagating in opposite directions are obtained by Eq. (2) by using the gradient and the wave velocity of the wire $c$.

$$y(x,t) = f(x - c,t) + g(x + c,t) \quad \cdots (1)$$

$$f(x - c,t) = \frac{1}{2} \{ y(x,t) - c \int \frac{dy(x,t)}{dx} dt \} \quad \cdots (2)$$

$$g(x + c,t) = \frac{1}{2} \{ y(x,t) + c \int \frac{dy(x,t)}{dx} dt \}$$

A measurement example of the wave separation is shown in Fig. 3 (vertical acceleration limited under 60Hz). The wave motions propagate on the contact wire in different directions (forward and backward).
3.2 Contact force fluctuation caused by wave motion of contact wire

If the displacement of the wave motion incident to the pantograph $y$ is expressed by Eq.(3) in the analysis model shown in Fig.4, the amplitude of contact force fluctuation $F_p$ caused by the incident wave is obtained by Eq.(4),

$$y = A \exp \left\{ \omega (t - x/c_t) \right\}$$  

$$|F_p| = (1 \pm \beta) \omega |Z|, \quad Z = \frac{1}{1/Z_i + 1/Z_p}$$

where $A$ and $\omega$ are the amplitude and the angular frequency of the incident wave, respectively; $\beta$ is the ratio of the moving speed of pantograph to the wave propagating velocity of contact wire; $Z_i$ and $Z_p$ are the mechanical impedance of contact wire and pantograph; and $i$ is the imaginary unit. In the right hand side of eq.(14), the sign "+" is used when the wave motion comes incident from the front of the pantograph, while "-" is used for the wave motion from behind. Eq.(4) means that the magnitude of the contact force fluctuation is proportional to the vertical velocity amplitude of the wave incident to the pantograph.

Fig.3 Measurement example of wave separation.
Fig. 4  Analysis model for contact force fluctuation caused by wave motion of contact wire.

In order to verify the above-mentioned theoretical analysis, we measured the wave motion of the contact wire incident to the pantograph and the contact force fluctuation at the same time. An actual catenary and pantograph are available for the experiment with the current collecting test equipment of R.T.R.I. In order to measure the contact force, load-cells and accelerometers are set under the contact strips of PS200A-type pantograph. A contact wire is only provided for the running experiment, in order to simplify observation of the wave propagation.

The result of the experiment in rigid support is shown in Fig. 5 (speed: 100km/h, \( \beta = 0.28 \)). "V12_F\_"V34_G\" indicate the vertical velocities of the wave motion passing through the measuring points (limited under 60Hz). It is found that the wave motion propagates forward (1)(2), reflects at the forward support point (3), comes incident to the pantograph (4), and repeats the reflection between the support point and the pantograph (5) \( \ldots \) (10). A large contact force fluctuation is caused when the wave motion comes incident to the pantograph. The measuring results agree well with theoretical calculation.
3.3 Generation of wave motion by unevenness of contact wire

As shown in Fig.6, we suppose that the contact wire is an infinite string which has the unevenness of a wave length $\lambda$ and one-side amplitude $B$. When a pantograph runs under the contact wire (speed, $v$), the amplitude of vertical velocity of the generated backward wave and forward wave $|V_1|$, $|V_2|$ and the angular frequency $\omega_1$, $\omega_2$ are obtained by Eq.(5) and Eq.(6). The contact force fluctuation caused by the unevenness of contact wire $F_{uw}$ is obtained by Eq.(7).
These equations indicate that the frequency and the vertical velocity amplitude of the forward wave increases when the pantograph speed becomes higher, and the forward wave is especially influential at high-speed. Concerning the relation between the unevenness of contact wire and the amplitude of generated wave motion, we have confirmed that the measurement results of fundamental experiments with low spring constant hangers agree well with the theoretical analysis. 

### 3.4 Reflection of wave motion at hanger point

As shown in Fig.7, we suppose an analysis model in which two infinite strings are combined with a spring-damper element. The complex reflection factor $\gamma_R$ (the ratio of amplitude of reflected wave $A_{1r}$ to that of incident wave $A_{1i}$) is obtained by Eq.(8), where $k$, $D$, $m_t$, and $m_m$ denote the spring constant, the damping factor and the mass of a hanger, respectively, while $\rho_l$, $\rho_m$, $T_l$, $T_m$ represent the line density and the tension of contact wire and messenger wire, respectively.
Fig. 7 Analysis model for wave reflection at hanger.

\[ \gamma_r = \frac{A_{ir}}{A_{ii}} = \frac{Z_h}{Z_t + Z_h} \quad \ldots \ldots (8) \]

\[ Z_h = i\omega m_t + \frac{1}{Z_m + i\omega m_m + \frac{1}{1}} \quad \frac{1}{D + k\lambda \omega} \]

\( Z_t \) and \( Z_m \) are the mechanical impedance of contact wire and messenger wire, respectively, shown in Eq. (9).

\[ Z_t = 2\sqrt{\rho_t T_t}, \quad Z_m = 2\sqrt{\rho_m T_m} \quad \ldots \ldots (9) \]

Fig. 8 is an example of the transfer coefficients of contact wire at a hanger point (heavy-compound catenary; between contact wire and auxiliary messenger wire). These are calculated by taking into consideration the line flexural rigidity. These coefficients are measured on a real overhead contact line by the wave separation measuring method and spectrum analysis method\(^6\). It is confirmed that the measured values are mostly in agreement with theoretical values, and the line flexural rigidity is influential on the transfer coefficient even at low frequencies.
4. Mechanism of contact force fluctuation in hanger span cycle

As shown in Fig.9, we suppose that the wave motion (1) is generated by the dip of contact wire between hangers, (2) reflects at the nearest hanger, (3) becomes incident to the pantograph and causes the contact force fluctuation.

Fig.8 Coefficient of wave transfer at hanger.

Fig.9 Analysis model for contact force fluctuation in a hanger span cycle.
The unevenness of contact wire is also measured on a real overhead contact line by the unevenness precise measuring method applied with the laser technique\(^7\). Fig.10 shows a measurement example of unevenness of contact wire of Shinkansen. The unevenness of wavelength \(\lambda = 5\) m (wave number \(k = 0.2\)) is equivalent to the dip between hangers. The shorter wavelengths are also seen in this measurement result.

Fig.10 Measurement example of contact wire unevenness in Shinkansen.

If the dip between hangers is assumed to be a secondary curve, the \(k\)-th coefficient of Fourier series \(a_k\) is expressed by Eq.\((10)\), where \(L_h\) is the span length between hangers.

\[
a_k = \frac{\rho g}{2T} \left(\frac{L_h}{\pi k}\right)^2 \cos(\pi k) \quad (k \neq 0)
\] .......(10)

The amplitude of the \(k\)-th unevenness is \(1/k^2\) times the primary unevenness, and its wavelength is \(1/k\) times. Therefore, it is shown from Eq.(6) that the vertical velocity amplitude of the contact wire wave motion generated by the \(k\)-th unevenness becomes \(1/k\) times in the case of primary unevenness. In other words, the influence of the higher order unevenness of the dip between hangers on the generation of contact wire wave motion is comparatively small.

If the contact wire unevenness can be regard as a first harmonic of the dip between hangers, the amplitude of contact wire unevenness \(B\) is simply expressed by Eq.\((11)\).

\[
B = a_1 = \frac{g}{2c_t^2} \left(\frac{L_h}{\pi}\right)^2
\] .......(11)

The vertical velocity amplitude and the angular frequency of the generated wave motion of contact
wire (forward wave) $|V_1|$, $\omega_2$ are obtained by Eq. (12), and the amplitude of the contact force fluctuation caused only by the unevenness of contact wire $|F_{ue}|$ is obtained by Eq. (13).

$$|V_1| = \frac{g}{\pi} \left| \frac{Z_p}{Z_t + Z_p} \right|_{\theta = \theta_b} \cdot \beta \frac{L_h}{1 - \beta c_i}$$

$$\omega_2 = 2\pi \frac{\beta c_i}{1 - \beta L_h}$$

$$|F_{ue}| = \frac{2 \rho g L_p}{\pi} \left| \frac{Z_p}{Z_t + Z_p} \right|_{\theta = \theta_b} \cdot \beta$$

$$\omega_0 = 2\pi \frac{c_i}{L_h}$$

Fig. 11 shows a measurement example on Shinkansen line in which the forward wave is generated by the dip of contact wire between hangers and propagates in front of the pantograph. Under the conditions of $\lambda = 5m$, $c_i = 114m/s$, and $v = 61.1m/s$ (220km/h), the frequency of the generated wave motion is calculated as 26Hz from Eq. (12). The frequency of the forward wave observed by this measurement is mostly in agreement with this calculated value. It is confirmed that the contact wire wave motion is also generated by the dip between hangers in actual equipment.

Next, if it is assumed that the forward wave motion reflects $R$ times at a hanger point, the vertical velocity amplitude and the angular frequency of the incident wave to a pantograph...
\[ V_3 = \omega_3, \text{ and the amplitude and the angular frequency of contact force fluctuation } |F_{wv}|, \omega_{wv} \text{ are obtained as follows.} \]

\[ |V_3| = \frac{g}{\pi} \sqrt{\frac{Z_p}{L_{zh}}}, \frac{\beta}{1 - \beta} L_h \]

\[ \omega_3 = \frac{2\pi c_t}{1 - \beta} L_h \]  

\[ |F_{wv}| = \frac{2\rho_1 g L_h}{\pi} \sqrt{\frac{Z_p}{Z_t + Z_p_{zh}}}, \frac{(1 + \beta) L_h}{1 - \beta} \]

\[ \omega_{wv} = \frac{2\pi (1 + \beta) c_t}{1 - \beta} L_h \]  

Fig.12 shows the contact force fluctuation caused by the contact wire wave motion \( F_{wv} \) presumed from the measurement result of the contact wire unevenness and residual diameter in the case of Shinkansen. The presumed contact force fluctuation by the contact wire wave motion \( F_{wv} \) is obtained by subtracting the contact force fluctuation by contact wire unevenness \( F_{ue} \) from the contact force fluctuation \( F_{ue} + F_{wv} \), which is equivalent to the residual diameter under the supposition that contact wire wear is proportional to the contact force. This example is presumed for the PS202-type pantograph when it runs at 200km/h. The tendency of the contact force caused by the contact wire wave motion \( F_{wv} \) becoming higher is regularly seen at the 0.7~1.0 part of hanger span. The phenomenon in which the generated contact wire wave motion repeats the reflection between a front reflecting point and a pantograph is observed by the above-mentioned fundamental experiment. In this case, the first and second positions \( x_1 \) and \( x_2 \) at which the contact wire wave motion becomes incident to a pantograph locate 0.72 and 0.92, respectively, in a hanger span. These points agree well with the point where the contact force fluctuation caused by wave motion \( F_{wv} \) is large in this Figure. This means that the wave reflection at the nearest hanger is most influential for the wave incident to the pantograph.

It has been confirmed that the contact loss in a hanger span cycle occurs near the center between hangers and that the residual diameter near the center between hangers is larger\(^{11}\). It is thought that these phenomena result from this mechanism. That is, the contact force fluctuation in a hanger span cycle is considered to be the phenomenon in which the contact force fluctuation by the dip of contact wire between hangers \( F_{ue} \) and that by the contact wire wave motion reflected at a hanger \( F_{wv} \) are compounded.
5. Contact force fluctuation in high-speed operation

5.1 Speed characteristic of contact force fluctuation in a hanger span cycle

Fig.13 shows an example of the amplitude of contact force fluctuation $|F_{ue}|$ and $|F_{wv}|$ which is calculated by Eq.(13) and Eq.(15). These curves show the characteristics of non-dimensional speed in the condition of heavy-compound catenary and $Z_p > Z_t$. The contact force fluctuation caused by contact wire unevenness $|F_{ue}|$ increases linearly with speed. On the other hand, when $\beta$ exceeds about 0.7, the contact force fluctuation caused by the reflective wave motion $|F_{wv}|$ increases remarkably. This characteristic is based on the so-called Doppler effect as shown in Eq.(15). Consequently, it is shown that the influence of wave reflection becomes larger especially in high-speed operation.

This characteristic of contact force fluctuation agrees well with the radically increasing tendency of contact loss ratio. Therefore, we think that the phenomenon, in which the contact loss ratio increases remarkably as the running speed approaches the wave propagation velocity of contact wire, is mainly based on the contact force fluctuation in a hanger span cycle.

The amplitude of contact force fluctuation caused by the reflective wave motion $|F_{wv}|$ is compared and shown in Fig.14 for the case of heavy-compound (HC) catenary and the case of high-tension heavy-compound (HTHC) catenary for which the tension of contact wire is increased to 19.6kN. In this case, these values are calculated with the parameter of PS202 pantograph. In the case of high-tension heavy-compound catenary, the wave propagation velocity of contact wire is about 15% higher, and the reduction effect of contact force fluctuation by decreasing $\beta$ is seen. Fig.15 shows the comparison of these cases at non-dimensional speed $\beta$, and it is found that these characteristics have almost the same curves. As shown in Eq.(15), this is because the amplitude of the contact force
fluctuation $|F_{wv}|$ mainly depends on the non-dimensional speed if the contact wire is the same type and the reflective coefficient is the same.

Fig.13 Calculated example of contact force fluctuation

Fig.14 Comparison of contact force fluctuation
By using Fig. 14, we will estimate here the maximum speed without contact loss. We suppose that the stationary uplift force of the moving pantograph is 74N (static uplift force 54N + aerodynamic uplift force 20N) and contact loss will occur when the amplitude of contact force fluctuation caused by wave motion reflection of contact wire exceeds the stationary uplift force. In regard to the speed dependence in this Figure, it is indicated that the maximum speed without contact loss (critical speed) is about 270km/h for the heavy-compound catenary, and 320km/h for the high-tension heavy-compound catenary. In the actual case, it is thought that contact loss occurs below these speeds because of other causes. However, the speed shown here becomes the standard value for the maximum operation speed as a contact performance index of overhead equipment. The high-tension heavy-compound catenary is one of the standard types for 300km/h high-speed operation in Japan.

5.2 Improvement methods of catenary for high-speed operation

Based on Eq. (15), improvement methods for high-speed operation are shown as follows.
(1) Increasing the wave propagation velocity of contact wire.
(2) Decreasing the density of contact wire.
(3) Reducing the wave reflection of contact wire.
(4) Shortening the hanger span length.

The present overhead equipment of Shinkansen for 300km/h operation has a high wave propagation velocity of contact wire by increasing the tension of contact wire, and adoption of lightweight contact wire. This is desirable from the viewpoint of this research. Moreover, if it has the same wave propagation velocity, the smaller the line density of contact wire is, the better the performance is.

In order to reduce the reflection of contact wire wave motion at a hanger point, use of hangers which have a spring and a damper mechanism is effective. Although the use of these damping type hangers cannot remarkably change the increasing tendency of contact loss depending on the relation between the train speed and the wave propagation velocity of contact wire, these hangers are expected
to reduce the contact force fluctuation below the critical speed. The reflection factor of wave motion with a damping hanger will be calculated as shown in Fig.16. A coil spring is used in the friction damping hanger, and a piece of rubber is used in the rubber damping hanger. From the results of the field test carried out on Sanyo Shinkansen line, it is found that there is a close correlation between the contact loss ratio and the vertical velocity of backward wave motion. The rubber damping hangers can reduce the magnitude of the wave motion incident to the pantograph and the contact loss ratio.

Fig.16 Reflection factor of wave motion at hanger.

The measurement results for Shinkansen lines show that the contact loss ratio in the section of a 3.5m-long hanger span is smaller than that in the section of 5m-long one. We think that our theory can explains such measurement results.

6. Conclusion

In the existing catenary equipment for Shinkansen lines, the contact force fluctuation in a support span cycle is comparatively small because it is a compound type and the whole tension is higher. Therefore, the contact force fluctuation in a hanger span cycle is comparatively large. From the above study, we have obtained the conclusions summarized below about the mechanism of contact force fluctuation and improvement methods for high-speed operation.

1. It is shown that the contact force fluctuation of pantograph in high-speed operation is mainly caused by the incident wave of contact wire that is generated by the unevenness of contact wire and is reflected at hangers.
2. This influence becomes extremely large when the train speed is close to the wave velocity of contact wire. The amplitude of contact force fluctuation exceeds the stationary uplift force of a pantograph at the non-dimensional speed of about 0.7 or over.
3. Increasing the wave velocity of contact wire or decreasing the reflection factor of wave motion at hangers effectively reduces the contact force fluctuation.
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