Correlation of vertical vehicle body acceleration and pantograph contact force

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Abstract:
The contact force of pantograph is one of the most important indexes of the current-collection performance. When contact force is lost dynamically, the phenomenon of contact loss occurs. Although many attempts, including analyzing the influence of dynamic contact wire wave conditions and the effect of unevenness of contact wire surface, have been done by a number of researchers previously on this topic, the relation between vertical vehicle body acceleration and contact force of pantograph has not been studied fundamentally. The study presented in this paper uses a pantograph model in consideration of the influence of vertical acceleration of vehicle body on the contact force fluctuation.

From calculation results, the influence of vertical vehicle body acceleration on the fluctuation of contact force was examined. From this result, it is pointed out that the vertical vehicle body acceleration in the low frequency region affects the contact force fluctuation of pantograph. The frequency response function of the vertical vehicle body vibration on the contact force was calculated by using the data measured in line tests. We estimated the fluctuation of contact force caused by the vertical vehicle body acceleration, and calculated the rate of contribution of the vertical vehicle body acceleration to the total fluctuation of contact force. Consequently, maximum about 30% of the total fluctuation of measured contact force are attributed to vertical acceleration of the vehicle body under 16Hz in the case of express lines.

Moreover, a comparison of calculation results and experimental results has shown that the frequency responses of the vertical vehicle body acceleration on the pantograph contact force are in good agreement. Hence, this method is considered to provide a useful technique for maintenance of overhead catenary system.

Keywords: electric railway, overhead catenary system, current collection, vibration, pantograph, contact force, vertical vehicle body acceleration

1. Introduction

The contact force of pantograph is one of the most important indexes of the current-collection performance. When contact force is lost dynamically, the phenomenon of contact loss occurs. Although many attempts, including analyzing the influence of dynamic contact wire wave conditions and the effect of unevenness of contact wire surface, have been done by a number of researchers previously on this topic, the relation between vertical vehicle body acceleration and contact force of pantograph has not been studied fundamentally. The study presented in this paper uses a pantograph model in consideration of the influence of vertical acceleration of vehicle body on the contact force fluctuation.
2. Calculation

In this chapter, the influence of the vertical vehicle body acceleration on the fluctuation of contact force is examined based on a model for the pantograph and catenary system.

2.1 Pantograph model

In using the pantograph model shown in Fig. 2, the equation of motion of the pantograph is calculated with considerations to the rotation of a links 2 and 3 and the vertical vehicle body acceleration. The equation of motion is drawn from the Lagrange's equation. In this model, the kinematic energy \( T \), the potential energy \( U \) and the dispersive energy \( D \) can be found as follows.

\[
T = \frac{1}{2} \left( m_1 \ddot{x}_1^2 + m_2 \left( \frac{x_2 + x_4}{2} \right)^2 + m_3 \left( \frac{x_3 + x_4}{2} \right)^2 \right) + \frac{1}{2} I_2 \ddot{\phi}_2^2 + \frac{1}{2} I_3 \dot{\phi}_3^2
\]  

(1)

\[
U = \frac{1}{2} k_1 (x_1 - x_2)^2
\]

(2)

\[
D = \frac{1}{2} c_0 \dot{x}_1^2 + \frac{1}{2} c_1 (\dot{x}_1 - \dot{x}_2)^2 + \frac{1}{2} c_2 (\dot{x}_2 - \dot{x}_3)^2
\]

(3)

where

\( x_1 \): the vertical displacement of pan heads
\( x_2 \): the vertical displacement of pantograph knees
\( x_3 \): the vertical displacement of vehicle bodies
\( \phi_1 \): the angle of pantograph upper frames
\( \phi_2 \): the angle of pantograph lower frames

The equation of motion of pantograph can be expressed by the following equation based on the equations (1) - (3).

\[
M \ddot{X} + C \dot{X} + KX = B
\]

\[
X = [x_1 \ x_2 \ x_3]^T
\]

(4)
The pantograph model in this paper differs in modeling a frame from the conventional model. Moreover, $m_{22}$, $m_{23}$, $m_{32}$ and $m_{33}$ in the preceding equations are coefficients calculated from the equivalent mass of the frame. Therefore, the conventional parameter cannot be used for these equations. Then, with a pantograph examination machine of R.T.R.I, an identification experiment (figure 3) was performed for the pantograph PS203 of Shinkansen, and a mass matrix and a dispersive matrix was identified. The identification examination was carried out only for the upper and lower frames of the pantograph. The top of the upper frame was fixed to the upper exciter through a force transducer, and the upper and lower exciters were excited independently. The result of the identification experiment is shown in Fig. 4.

The catenary model is considered as an infinite string and reflection of the wave motion produced by the fluctuation of contact force is not taken into consideration. Furthermore, the frequency range is limited only to low frequency because it is assumed that the catenary moves as one line. Therefore, it is considered that the total linear density and the total tension are the linear density and the tension of this catenary model, respectively. Although train velocity influences the mechanical impedance of the catenary, this effect is not taken into consideration.

As mentioned above, the catenary is considered to be a damper $c_0 = \sqrt{\sum \rho \sum T}$ equivalent to a mechanical-impedance. The contact force of a pantograph $f_0$ is calculated as $f_0 = c_0 \cdot \lambda_0$. 

\[
B = \begin{bmatrix}
0 & 0 & f_3 \\
0 & 0 & 0 \\
m_1 & 0 & 0 \\
0 & m_{22} & m_{23} \\
0 & m_{32} & m_{33}
\end{bmatrix}
\]

\[
M = \begin{bmatrix}
c_0 + c_1 & -c_1 & 0 \\
-c_1 & c_1 + c_2 & -c_2 \\
0 & -c_2 & c_2
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
k_1 & -k_1 & 0 \\
-k_1 & k_1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]
Fig. 2 Model of pantograph
Fig. 3  Excitation experiment facility

Fig. 4.1  Identification result
2.2 Calculation results for a Shinkansen line

By using the coefficient obtained from the identification examination in Section 2.1, the frequency response is calculated. The calculated frequency response to the contact force by the vertical vehicle body acceleration is shown in Fig. 5. Moreover, the spectrum of the vertical vehicle body acceleration measured in the Shinkansen line test is shown in Fig. 6. The spectrum of the predicted fluctuation of contact force by the vertical vehicle body acceleration calculated from Fig. 5 and 6 is shown in Fig. 7. This result shows that the pantograph contact force is influenced by the vertical vehicle body acceleration in the low frequency range, in particular.

The parameter of the catenary used in this calculation is shown in Table 1. The coefficient of the pantograph identified from the excitation examination is also shown in Table 2.
Fig. 5 Frequency response of contact force against vertical vehicle body acceleration

Fig. 6 Spectrum analysis results of vertical vehicle body acceleration measured in the line test
Fig. 7 Spectrum analysis results of predicted contact force

<table>
<thead>
<tr>
<th></th>
<th>Linear Density (kg/m)</th>
<th>Tension (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact wire</td>
<td>1.511</td>
<td>14.7</td>
</tr>
<tr>
<td>Auxiliary messenger wire</td>
<td>1.375</td>
<td>14.7</td>
</tr>
<tr>
<td>Messenger wire</td>
<td>1.450</td>
<td>24.5</td>
</tr>
</tbody>
</table>

Table 1 Parameter of catenary

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>$m_{12}$</td>
<td>8.3 [kg]</td>
</tr>
<tr>
<td>$m_{23}$</td>
<td>4.0 [kg]</td>
</tr>
<tr>
<td>$m_{32}$</td>
<td>4.0 [kg]</td>
</tr>
<tr>
<td>$c_0$</td>
<td>966.9 [Ns/m]</td>
</tr>
<tr>
<td>$c_2$</td>
<td>59.5 [Ns/m]</td>
</tr>
</tbody>
</table>

Table 2 Parameter of pantograph
3. Experimental results

In Chapter 2, the influence of vertical vehicle body acceleration on the pantograph contact force was predicted from model calculation. In this chapter, the line test data are analyzed.

3.1 Correlation of vertical vehicle body acceleration and pantograph contact force

The coherence function \( \gamma_{x_f}^2 \) of vertical vehicle body acceleration \( x \) and the pantograph contact force \( f \) is calculated from the equation (5) by using the data measured on Shinkansen and express lines. The results are shown in Fig. 8. Here, \( W \) represents a power spectrum or a cross spectrum. In addition, averaging has been performed 200 times.

\[
\gamma_{x_f}^2 = \frac{W_{x_f} W_{f_x}}{W_{f_f} W_{x_x}} \tag{5}
\]

In the case of an express line, Fig. 8 shows that coherence function is maximum about 0.2 in under 2Hz. On the other hand, in the case of Shinkansen, coherence function is about maximum 0.1 under 2Hz. This difference is considered that the vertical vehicle body acceleration is small in the case of the Shinkansen. Here, the whole fluctuation of contact force is denoted \( f \); the fluctuation of contact force by the body acceleration is denoted \( f_a \); the fluctuation of contact force by other factors is denoted \( n \). These relations are shown by the following equation.

\[
f = f_a + n \tag{6}
\]

Here, when it is assumed that there is no correlation between \( n \) and \( x \), it can statistically be considered as \( W_{x_f} = W_{x_f a} \) and the frequency response \( H_1 \) is shown by the following equation (7).

\[
H_1 = \frac{W_{x_f a}}{W_{x_x a}} = \frac{W_{x_f}}{W_{x_x}} \tag{7}
\]

In the case of an express line, the frequency response to the pantograph contact force of the vertical vehicle body acceleration calculated from the line test data by using the equation (7) is shown in Fig. 9. The results show that the contact force is influenced by the vertical vehicle body acceleration especially in the low frequency range. The same tendency was also obtained in previous model calculation. In addition averaging was carried out in 2000 times in calculating a frequency response.
Fig. 8  Coherence between contact force and vertical vehicle body acceleration
3.2 Prediction of contact force depending on vertical vehicle body acceleration in Time History

In this section, the time histories of the predicted fluctuation of contact force $f_a$ by the vertical vehicle body acceleration are calculated. They are the time histories of train velocity, vertical vehicle body acceleration, and the contact force (the total contact force and the contact force caused by the vertical vehicle body acceleration) from the upper row in Fig. 10. The calculation method of the predicted fluctuation of contact force by the vertical vehicle body acceleration is shown below. First, the impulse response function $h(t)$ is calculated from the frequency response $H_1(\phi)$ in Fig. 9 by using the equation (8).

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_1(\phi)e^{j\phi t}d\phi$$  \hspace{1cm} (8)

The fluctuation of contact force caused by the vertical vehicle body acceleration is also calculated from the impulse response function of the equation (8) and the time history of the vertical vehicle body acceleration by using the equation (9).

$$f_a(t) = \int_0^t \tilde{x}_3 h(t - \tau) \tau$$ \hspace{1cm} (9)

It is predicted from Fig. 10 that the fluctuation of contact force caused by the vertical vehicle body acceleration is maximum about 10N. Even when the coherence is low, it is thought from this result that the frequency response can be predicted by repeating averaging.
3.3 Influence of vehicle body vertical acceleration on contact force

By using the data of the fluctuation of contact force by the vertical vehicle body acceleration calculated in Section 3.2, the rate of contribution of the vertical vehicle body acceleration at every train velocity $v$ is calculated. The result is shown in Fig. 11. Fig. 11 shows the RMS value of the total fluctuation of contact force $f_{rms}(v)$, the RMS value of vertical vehicle body acceleration $a_{rms}(v)$, the RMS value of the predicted fluctuation of contact force caused by the vertical vehicle body acceleration $f_{a rms}(v)$, and the rate of contribution to the contact force of the vertical vehicle body acceleration $r_{fx3}(v)$ from the upper row. In addition, the equation (10) is used to calculate $r_{fx3}(v)$. 

Fig. 10 Time-history of contact force
However, Fig. 11 is the result of evaluating the signals under 16Hz. In the high frequency range, the power of vertical vehicle body acceleration and the gain of the frequency response of the fluctuation of contact force caused by the vertical vehicle body acceleration are small. Therefore, if high frequencies are taken into consideration, it is thought that the contribution of the vertical vehicle body acceleration to the total fluctuation of contact force becomes smaller. In the case of this express vehicle, the rate of contribution of the vertical vehicle body acceleration to the fluctuation of contact force is the maximum at about 50km/h, and accounts for about 30% on an average of the total fluctuation of contact force under 16Hz. However, this result depends on the characteristics of the vehicle, pantograph, and catenary system.

\[
rf_{x_3}(v) = \frac{f_{a,rms}(v)}{f_{rms}(v)}
\]
3.4 STFT in vertical vehicle body acceleration and contact force

The short-time Fourier transform (STFT) shown in the equation (11) has been performed for the data measured on an express line as shown in Fig. 12.

\[
(G_{\beta} f t \theta) = \int_{-\infty}^{\infty} e^{-j \omega t} f(t) g_{\alpha}(t - \theta) dt
\]  

(11)

Fig. 12 shows the time histories of contact force, vertical vehicle body acceleration, train velocity, STFT results for contact force, STFT results for vertical vehicle body acceleration, and the coherence function. The intensity of the result of STFT is normalized on a log scale except for the coherence function. And the intensity of the result of STFT in coherence function is normalized on a linear scale. In addition, the Gaussian window, shown in the equation (12), is used in STFT.

\[
g_{\alpha}(t) = \frac{1}{2\sqrt{\pi}\alpha} e^{-t^2/4\alpha}
\]  

(12)

Fig. 12 is the result for the section where a train runs at 50km/h - 120km/h. According to the train velocity, it turns out that the frequency and the intensity of the contact force and the vertical vehicle body acceleration change. Although the center of the time-axis is the section where the train runs at 50km/h, the coherence function of the contact force and the vertical vehicle body acceleration become high at 1.5Hz. In Section 3.3 it is shown that the rate of contribution of the vertical vehicle body acceleration to the contact force is high at 50km/h.

It is thought that these are the same phenomena. The following two factors can be considered in these phenomena.

(1) Since the train velocity is low, the fluctuation of contact force by factors other than the vertical vehicle body vibration is small.

(2) The 1.5Hz signal is strong at 50km/h from the STFT result of the vertical vehicle body acceleration. The reason is that the natural frequency of the vertical vibration of these vehicles is 1.5Hz, and that one of the excitation frequencies depending on the train velocity is in 1.5Hz at 50km/h, and this excitation frequency is in agreement with the natural frequency of vehicles.

For the above reason, it is thought that the rate of contribution by the vertical vehicle body acceleration to the total fluctuation of contact force became the maximum in Fig. 11 when the train runs at 50km/h.
Fig. 12 STFT results of contact force and vertical vehicle body acceleration
3.5 Comparison of calculation results and experimental results

Fig. 13 shows the two frequency response curves of the contact force by the vertical vehicle body acceleration of Shinkansen, which have been calculated from the calculation result of Section 2.2 and from the line test result. In general, although it is thought that they are in agreement, they differ from each other as frequency goes up. This is caused by the assumption that catenaries vibrate as one infinite string in modeling. Moreover, it is an error factor that the vertical vehicle body acceleration of Shinkansen is comparatively small.

4. Conclusion

An outline of conclusion of this paper is as follows.

(1) The influence of the vertical vehicle body vibration on the fluctuation of contact force has been investigated. This result suggests that the contact force of pantograph is influenced by the vertical vehicle body acceleration especially in the low frequency range.

(2) A method to calculate the frequency response of the vertical vehicle body acceleration to the fluctuation of contact force has been proposed, from the data obtained from line tests.

(3) By using the frequency response of the vertical vehicle body acceleration to the
fluctuation of contact force, the rate of contribution of vertical vehicle body acceleration to the total fluctuation of contact force has been calculated. Consequently, it may become to be 30% under 16Hz when an express train runs.

(4) A comparison of experimental results and calculation results has been performed about the influence of the vertical vehicle body acceleration on the fluctuation of contact force. In general, although it is thought that they are in agreement, they differ from each other as frequency goes up. This point is a future subject.

Finally, based on the comparison between the experimental results and calculation results in Section 3.5, a method to conversely identify a catenary model can be considered. Since it is thought that this method is useful for the maintenance of a catenary, we will advance this research in the future.

Reference