Abstract
The contact force between pantograph and contact wire is an essential technical quantity to determine the dynamic interaction of pantograph with the overhead catenary system. The contact force is closely related with the wear of contact wire and contact strips. Precise measurement of the contact force is needed for good assessment and maintenance of the overhead catenary system, and becomes an important issue regarding good performance of current collection system.

By using the methods currently used, the contact force can be measured in the frequency range up to 20~40Hz. Above this frequency, measurement results contain large errors. The present methods treat the panhead as a simple rigid body. This leads to large errors in the measurements in the high frequency range where elastic vibration modes of the panhead dominate. On the other hand, the frequency of the contact force fluctuation caused by the wave propagation in the contact wire is above 20Hz in the high-speed region. Some theoretical studies show that the current collection performance mainly depends on this contact force fluctuation for high-speed trains.

The newly proposed measuring method can evaluate the inertia force more precisely. This method is based on the vibration mode superposition, and estimates the inertia force by adding accelerations at multi-points of each panhead with adequate weights. Hence, precise measurements can be obtained in the higher frequency range where elastic vibration of the panhead dominates.

Practically, in the case of the standard pantograph of Shinkansen train, the contact force up to 100 Hz can be measured by using three accelerometers at each panhead. The new method has high accuracy in regard to the gain and phase characteristics, and an advantage for high quality maintenance of overhead catenary system. As this method requires only a simple configuration of sensors, it can be an appropriate and practical method for various types of pantographs.

Key word: Current collection, Contact force, Pantograph, Overhead catenary system, Measurement
1. Introduction

The contact force between pantograph and contact wire determines the dynamic performance of current collection. Large contact force furthers wear of contact wire and also causes fatigue of the wire. On the other hand, too small contact force gives rise to arcing. Therefore, keeping contact force in a suitable range is very important for contact wires and contact strips to have a long life.

To evaluate the dynamic performance of current collection, the contact force has widely been measured in the world railways. But the contact force can be measured in the frequency range up to 20~40Hz by the currently used methods\(^1\)\(^2\). Above this frequency, the measurement results contain large errors. In general, such errors are ascribed to the incorrect compensation of inertia force acting on the panhead. The present methods treat the panhead as a simple rigid body. This leads to large errors in the high frequency range where elastic vibration modes of the panhead dominate. In other words, by the present methods, the contact force can be measured precisely only in the frequency range sufficiently lower than the natural frequency of the first elastic mode of the pantograph.

We can measure the contact force on the wayside by using the super-anamorphic-lens with focal length ratio of about 80:1\(^5\)\(^6\). This lens permits simultaneous televiewing of contact wire system over several spans. The contact force can be estimated by the gradient of contact wire near the pantograph. This feature is very useful, because measurement on the car is not necessary. But this method is unsuitable for continuous measurement with high precision.

Some theoretical studies\(^3\)\(^4\) show that the current collection performance of high-speed train mainly depends on the contact force fluctuation caused by dips between hangers. The frequency of this fluctuation is above 20Hz in the high-speed range. Moreover, exact extraction of contact loss needs a precise contact force measurement at widely ranged frequencies. So we have developed a new measurement method.

Of course, dynamic characteristics of a pantograph equipped with sensors must not differ from those of original one. As a matter of course, reconstruction of a panhead for the measurement has to be limited. We propose a new measuring method that can make precise evaluation in the higher frequency range than the present methods can. The key technology is the evaluation method for inertia force by using plural accelerometers. This is based on the superposition of vibration modes.

2. Principle of contact force measurement method

In general, the contact force can be obtained by measuring the forces acting on suitable cross-sections of the panhead and inertia force of the panhead. In order to explain this, a pantograph is modeled as shown in Fig. 1. The pantograph is divided into the subsystem 1 and the subsystem 2. The cross-sectional force acting on a connecting point between subsystem 1 and subsystem 2 is defined as \(F_{d,i}\). The contact force \(F_c\) shall be indicated statically as follows.

\[
F_c = \sum k_i F_{d,i} \tag{2.1}
\]

Where \(k_i\) is a proportional constant.

Dynamically, the contact force is given by the equation (2.2).

\[
F_c - F_{aero} = \sum k_i F_{d,i} + F_{inc} \tag{2.2}
\]

Where \(F_{aero}\) is the aerodynamic force acting on the subsystem 1, and \(F_{inc}\) is the inertia force of the subsystem 1. The sign of these forces are defined as shown in Fig.1. Therefore, we can estimate the contact force by measuring the inertia force of the subsystem 1, cross-sectional forces at the connecting points between the subsystem 1 and the subsystem 2, and the aerodynamic force on the subsystem 1. It should be noted that we have to sum up all cross-sectional forces between two subsystems for the equation (2.2). Unless we do so, the equation (2.2) cannot hold exactly independent of the contact position. Direct measurement of aerodynamic force is very difficult, so we usually evaluate \(F_c - F_{aero}\) as the contact force measurement.
3. New measuring method for contact force

We propose a new contact force measuring method in this chapter. All measuring methods currently used have the same problem. It is that measurement errors become large in the high frequency range. These errors depend on the measurement precision of the inertia force. So far it is common that the inertia force of the subsystem 1 is measured by only one accelerometer. In the high frequency range, the vibration of the subsystem 1 cannot be regarded as a simple rigid body motion. That is to say, vertical acceleration at any point in the subsystem 1 is not the same in this frequency range. Therefore, estimation of inertia force by only one accelerometer becomes inaccurate.

To avoid this, we have to select the subsystem 1 so that its first elastic mode has a high natural frequency. Usually, a contact strip is selected as the subsystem 1 for this reason. This selection has an additional advantage that the aerodynamic force acting on the subsystem 1 is small. But the panhead must be reconstructed in order to install force measurement sensors under contact strips. Therefore, this method cannot be used in wide range.

Then we propose an improved method to estimate the inertia force. This method uses acceleration of plural points on the subsystem 1. When it is assumed that internal damping of the subsystem 1 can be neglected, the inertia force of the subsystem 1 is expressed in the form of the superposition of up to n'th vibration modes by the following formula.

\[ F_{\text{ine}}(t) = - \sum_{i=1}^{n} \int \rho(x) \ddot{y}_i(t) \nabla_i(x) dV \quad (3.1) \]

where \( x \) is the position vector, \( \nabla_i(x) \) is the modal coordinate of the \( i \)th mode, \( \dot{p}_i(t) \) is the modal displacement of the \( i \)th mode, \( (x) \) is density. Suppose that \( F_{\text{ue}} \) can be expressed as a weighted summation of accelerations at \( n \) discrete points \( x_j \) \((j=1 \sim n)\). That is,

\[ F_{\text{ue}}(t) = - \sum_{j=1}^{n} w_j \sum_{i=1}^{n} \ddot{p}_i(t) \nabla_i(x_j) \quad (3.2) \]

where \( w_j \) is the weight coefficient of acceleration at the point \( x_j \). If we make the equation (3.1) equal to the equation (3.2), the equation (3.3) is given as

\[ \sum_{i=1}^{n} \ddot{p}_i(t) \int_{\text{subsystem1}} \rho(x) \nabla_i(x) dV - \sum_{j=1}^{n} w_j \nabla_j (x_j) = 0 \quad (3.3) \]

The necessary and sufficient condition for the equation (3.3) to hold with arbitrary \( \ddot{p}_i(t) \) is derived as follows.
This equation is the \( n \)'th simultaneous equation in which unknown parameters are \( w_j \) and known parameters are the distribution of mass density and modal coordinates. We can obtain the weight coefficient \( w_j \) by solving the equation (3.4).

If the case where \( i=j=1 \) is considered in the equation (3.4), the following relationship holds.

\[
\int_{\text{subsystem } 1} \rho(x) \, dV = w
\]  

This equation means that the weight coefficient is equal to the whole mass of the subsystem 1. This case is equivalent to the method currently used. Therefore, we can measure in the higher frequency range in consideration of elastic modes by using more accelerometers.

4. Sensor configuration of pantograph for measurement

4.1. Sensors

In this chapter, we show an example of this measuring method applied to a Sinkansen pantograph. This pantograph is the type 203 that is used for Nozomi express trains. It is a diamond type pantograph with two panheads.

![Balance of forces acting on the panhead](image.png)

Two cross-sections A and B are set outside the main contact strips as shown in Fig. 2. The shear force acting on each cross-section is described as \( \tau_A \) and \( \tau_B \), respectively. When the pantograph contacts a contact wire between the cross-sections A and B, the contact force \( F_c \) is expressed by the equation (4.1) when it is assumed that the panhead can be modeled as an uniform beam with constant line density \( \gamma \).

\[
(\tau_A - \tau_B) - \int_{l_A}^{l_B} \rho \frac{\partial^2 \gamma}{\partial x^2} \, dx = F_c \]  

where \( \gamma(x,t) \) is the vertical displacement of the panhead at the position \( x \); \( l_c \) indicates the contact position of a contact wire; \( l_A \) and \( l_B \) indicate the positions of cross-sections A and B, respectively. Here we do not take into consideration the aerodynamic force.

We can regard the section between the cross-sections A and B as the subsystem 1 and the section outside the cross-sections A and B as the subsystem 2. Then the first term of the left hand side of the equation (4.1) is equivalent to the sum of all cross-sectional forces acting on the connecting points of the subsystem 1 and 2. That is to say, the shear force can be chosen as a cross-sectional force. The second term is equivalent to the inertia force of the subsystem 1.
Fig. 3 shows the sensor arrangement on the panhead for measurement of contact force. We measure the shear force of the panhead by two 2-axis (90 degrees) strain gages at both sides (forward side and backward side) of the panhead. The two strain gages form a four-gage bridge circuit by which we can obtain the shear force at the gage-attached point. By this way, the torsional distortion that occurs by the contact force in the running direction does not influence the shear force measurement. In addition, we use less-inductive type strain gages which can perform well under a strong magnetic field, because the contact force is measured by using the pantograph collecting current.

The number of necessary accelerometers depends on the available measurement frequency range. Here, our target is set up to 100Hz. The influence of the aerodynamic force is relatively large in above 100Hz, so the measurement of the aerodynamic force should be indispensable. Fig. 4 shows the measurement result of a transfer function between contact force and acceleration of the panhead at the center position. We can observe a keen peak near 80Hz. This peak indicates the natural frequency of the first elastic mode of the panhead. Below this frequency, we can point out other peaks. These are rigid vibration modes. That is, they are a vertical translation mode and a rolling mode. Therefore, we must consider three modes (first elastic mode, vertical rigid mode, and rolling mode) up to 100Hz. Then three accelerometers are required to precisely estimate the inertia force. We attach three accelerometers under each panhead as shown in Fig. 3.

Moreover, we can estimate the position of the contact point on the panhead which is equal to a zig-zag position of contact wire by using two shear forces. It is because the following relation holds quasi-statically.

\[
\frac{\tau_A}{-\tau_B} = \frac{l_c - l_B}{l_A - l_c}
\]

(4.2)
4.2. Method of determining weight coefficients

To determine the weight coefficients of three accelerometers, we must solve the equation (3.4) analytically. But the estimation of mode shapes is complicated. Here we show a brief method to use experimental data.

First, we replace the inertia force with a weighted summation of three accelerations in the equation (4.1). Then following equation is obtained.

$$\tau(t) = \sum_{i=1}^{3} w_i \gamma(x_i, t) - F_c(t)$$  \hspace{1cm} (4.3)

where equals A B. The frequency response functions of the shear force and the acceleration against the contact force are described as $H_s$ and $H_a$, respectively. Then the equation (4.3) can be rewritten in the frequency domain as follows.

$$H_s(\omega) = \sum_{i=1}^{3} w_i H_a(\omega) = 1$$  \hspace{1cm} (4.4)

Since the accelerometer 2 and the accelerometer 3 are attached symmetrically as shown in Fig. 3, the following equation can be assumed.

$$w_2 = w_3$$  \hspace{1cm} (4.5)

In the low frequency range where the translation mode is dominate, the output of three accelerometers must be equal. Therefore, the sum of three weight coefficients should be equal to the mass of the subsystem 1. Then

$$\sum_{i=1}^{3} w_i = \rho(t_a - t_b)$$  \hspace{1cm} (4.6)

Under these conditions, three weight coefficients can be defined by only one independent parameter.

We can obtain the frequency response functions $H_s$ and $H_a$ by performing an exciting test. This test must be performed at some exciting points. By using the least square method, we can determine the weight coefficients for the equation (4.4) to be satisfied in the measurement frequency range.
5. Valuation of measuring precision

In this chapter, we valuate the measuring precision of this method. Fig. 5 shows the measurement result of the static contact force acting on the contact strip. In this Figure, the origin of the contact point indicates the center of the panhead. We can observe that the contact force can be estimated precisely independent of the contact position.

Dynamic valuation is done by a test using an exciter. We can measure the contact force exactly by a load cell on the tip of the exciter. At the same time, we also measure the contact force by the proposed method using sensors on the panhead. By comparing these results, we can obtain the measuring precision. Only one of a pair of panheads is excited at a time. We use quasi-random waves of 5~500Hz as an exciting signal.

Fig. 6 shows the time history of measured contact force. The top wave form is measured by the load cell, so we regard this as a true value. The middle one is the result estimated by the currently-used method by using only one accelerometer and strain gages. The bottom one is the result estimated by the new method using three accelerometers and strain gages. This Figure shows that the precision of the measurement with three accelerometers is higher than that with only one accelerometer.
Next, we estimate the measurement precision in the frequency domain. The estimation results are shown in Figs. 7-8. These Figures show the measurement precision in two cases. In the first case, the contact point between the pantograph and the exciter is exactly the center of the panhead. In another case, the contact point is 200mm distant from the center of the panhead. If the measurement precision is high, the gain should be unity and also the phase delay should be zero in these Figures.

In the case of the currently-used method, large errors can be observed above 40Hz. Especially large errors occur near 80Hz which is the natural frequency of the first elastic mode of the panhead. Furthermore, the measurement accuracy above 40Hz depends on the contact point. Therefore, the available frequency range of this method is recognized as below 40Hz.

On the other hand, the new method can measure the contact force precisely up to 100Hz. Even if the contact point changes, the precision of this measuring method does not become low.
6. **Line test**

We performed the contact force measurement on a Shikansen train by using this new method. Fig. 9 is the measurement result of the contact force compared with contact loss at 210km/h. The position where arcing is generated coincides with the position where the contact force is nearly zero. This means that the measurement method is sufficiently reasonable.
Fig. 9  Measured contact force compared with contact loss (210km/h)

Fig. 10  Measurement result of contact force at a simple catenary section

Fig. 10 shows the measurement result of the contact force at a simple catenary section. The height of the pantograph, the position of masts, and the zig-zag of contact wire are also measured at the same time. We can point out a typical feature of the simple catenary, which is the contact force fluctuation generated periodically in a support span cycle. But we can observe that the contact force fluctuation at specific sections becomes half amount of that at other sections in this Figure. As a matter of fact, friction-damping hangers are put to use instead of normal type hangers in this section. By a former research\(^3\), it was found that friction-damping hangers make the periodic fluctuation of the contact force in a support span cycle smaller. We can prove this theory by this measurement result.

Fig. 11 shows a short-time Fourier analysis result of measured contact force at a heavy compound catenary section. Short-time Fourier analysis is advantageous for the purpose to observe the dependence of wave fluctuation on time and position at the same time. The middle of Fig. 11 expresses the power spectrum density of contact force as a shade. The power spectrum density at the dark part is high. We can easily observe that distinguished frequencies exist. These frequencies change corresponding to the train velocity. Therefore, it may be predicted that these fluctuation of contact force are attributed to the geometric configuration of the catenary equipment.

The frequency axis of the short-time Fourier analysis is normalized by train velocity to clarify this prediction. In other words, the fluctuation is transferred from the frequency domain into the wave number domain. The bottom of Fig. 11 indicates this result. It is clearly observed that distinguished fluctuations exist at some specific wave numbers. The wave number of most notable contact force
fluctuation is 0.2. This is equivalent to the interval of hangers (5m). Subsequently, the fluctuation at the wave number 0.1 is large. This is equivalent to the interval of droppers (10m). From these results, it is sufficiently understood that the contact force fluctuation strongly depends on the periodicity of configuration of catenary equipment.

![Graph](image1.png)

![Graph](image2.png)

![Graph](image3.png)

**Fig. 11** Short-time Fourier analysis of the contact force at a heavy compound catenary section

7. **Conclusions**

We propose a new method of measuring contact force precisely up to a certain high frequency. This method consists of the estimation of inertia force by using of plural accelerometers and measurement of the cross-sectional forces acting on the suitable points.

In order to apply this method to the type PS203 pantograph of a Shinkanse train, strain gages are attached to measure shear forces as the cross-sectional forces, and also three accelerometers are put on
each panhead to estimate the inertia force. By an exciting test of the pantograph, it has been clarified that sufficiently precise measurement is possible up to 100Hz for this pantograph. Furthermore, the contact force measurement on a Shinkansen line was carried out, and the result shows that this method can be a powerful tool to estimate the performance of current collection. This method is easily applicable to different types of pantographs. We have already applied this method to some kinds of single-arm type pantographs. These measurement results show that this method is useful for many types of pantographs.

However, we have to develop a method to estimate the aerodynamic force acting on the panhead to improve the precision of measuring the contact force. The technology to estimate the conditions of the equipment of catenary system by measuring the contact force is also important as a maintenance technology. These are next targets of our research.

Bibliography

1) M.Ostermeyer, E.Dorfler; Die Messung der Kontaktkraft zwischen Fahdraht und Schleifleiste, Elektrische Bahnen 80, No.2, 47/52, 1982
2) Pantograph/overhead line interaction (Final report), ERRI(A186/RP10), 1996
4) Aboshi, M., Manabe M.; Analyses of contact force fluctuation between centenary and pantograph, Quarterly Report of RTRI, Vol. 41, No. 4, pp182-187, 2000